Financial Analysts Journal
Volume 66 • Number 6
©2010 CFA Institute

Performance Attribution: Measuring Dynamic Allocation Skill

Jason C. Hsu, Vitali Kalesnik, and Brett W. Myers

Classical performance attribution methods do not explicitly assess managers' dynamic allocation skill in the factor domain. The authors propose a generalized framework for performance attribution that decomposes the allocation effect into value added from both static and dynamic factor exposures and thus yields additional insight into sources of manager alpha.

At the end of 2008, there were more than 8,000 mutual funds in the United States, with $10 trillion in assets under management (AUM). Of these funds, approximately 80 percent were actively managed.[1] In the same year, around 9,000 U.S. hedge funds had $3 trillion in actively managed assets. Although some studies have cast doubt on the ability of the average active manager to consistently "beat the market," investors are nevertheless willing to entrust considerable sums of money to active management. Therefore, identifying skilled active managers is an important goal for many investors.

The last few decades have seen the introduction of numerous evaluation methodologies that characterize portfolio performance and identify talented active managers. The simplest of these methodologies is return-based regression analysis, such as the one predicated on the Fama and French (1992, 1993) three-factor model. With only return information, regression analysis can identify style tilts and estimate risk-adjusted alphas for managers. But although regression analysis requires few inputs, it also provides limited insight into sources of managerial performance.

Holdings-based attribution can provide a more detailed analysis of manager performance, but the input required is significantly more substantial than that required by standard regression analysis. Originally proposed by Brinson and Fachler (1985) in their study of manager skill in allocating to different industries, holdings-based attribution analysis has been extended to the study of allocation skill in other factor domains, such as value, size, momentum, and volatility. An industry standard, the so-called Brinson attribution analysis provides a straightforward way to decompose manager added value into such dimensions as superior factor/sector allocation and security selection.

Classical Brinson attribution was designed to analyze manager returns over a single period under the assumption of static holdings. It has since been extended to cover multiple periods to account for changing portfolio weights over the span of analysis.[2] Commonly used multiperiod attribution analyses, however, do not explicitly measure a manager's ability to allocate dynamically in the factor domain. This deficiency is important for a number of reasons.

For example, value stocks have historically outperformed growth stocks. A particular manager may seek to exploit this apparent value premium to generate a higher return against his benchmark by increasing the portfolio weights in value stocks. We term this approach *static factor allocation*, and the resulting alpha arises from persistent style tilts toward factors with a risk premium. Another manager, skilled in forecasting whether value stocks will outperform growth stocks in a given year, may dynamically adjust the value/growth tilt in her portfolio by increasing the weights in value stocks when she believes value will do well relative to growth, and vice versa. We term this approach *dynamic factor allocation*.

Although the sources of added value for these two managers are markedly different, traditional multiperiod Brinson-type analyses do not explicitly distinguish between them. The existing methods thus provide an incomplete assessment of a portfolio manager's investment style. In this article, we outline a methodology that decomposes the allocation effect of traditional attribution analyses

Jason C. Hsu is an adjunct professor at the UCLA Anderson School of Management and chief investment officer at Research Affiliates, LLC, Newport Beach, California. Vitali Kalesnik is a vice president at Research Affiliates, LLC, Newport Beach, California. Brett W. Myers is assistant professor of finance at the Rawls College of Business at Texas Tech University, Lubbock.

into static and dynamic components in a straightforward and intuitive way to enhance the measurement of manager skill.

A number of attribution studies in the academic literature have examined a manager's ability to adjust portfolio exposures dynamically. Notably, Daniel, Grinblatt, Titman, and Wermers (DGTW 1997) decomposed fund returns into the components *average style*, *characteristic selectivity*, and *characteristic timing* relative to a characteristic-based benchmark. Our attribution analysis is similar in spirit to the DGTW approach in that it seeks to characterize the manager's dynamic allocation skill in the factor domain. It is different, however, in that it accomplishes the analysis within the classical Brinson framework and does not require the added complexity of creating various characteristic-based benchmarks whose holdings must then be matched to each stock in the mutual fund in every period. Consequently, the methodology we present here is significantly less complex than the DGTW approach and more in line with current industry standards for performance attribution.

Motivation

Let us start with a simple example to introduce the intuition behind the attribution methodology we propose. Suppose that a balanced fund manager allocates his portfolio between two assets: equities (via an index fund) and cash. The equity market is expected to generate a positive return but with some variance. For simplicity, let us assume that cash bears zero interest and has no return variance. Suppose further that this manager attempts to time the equity market. When he believes the market will do well, he allocates away from cash and invests a fraction of the portfolio in the stock market; otherwise, he invests 100 percent in cash. Over time, the arithmetic average of his returns in each period can be written as

$$E\left(w_t R_t\right) = \frac{1}{T}\sum_{t=1}^{T} w_t R_t, \tag{1}$$

where w_t is the portfolio weight he places in the stock market at time t and R_t is the return on the stock market realized between t and $t+1$ (the weight in the cash account, $1 - w_t$, always returns zero).

After a time, our manager develops a history of delivering a positive return, on average, and interprets this record as evidence that he has successfully timed the market. There are at least two explanations, however, for his positive excess return. First, he may well have been successful in timing the market. Perhaps his return was garnered from market exposure during a limited number of days when the market did particularly well, and he managed to have low market exposure during periods of negative returns. In that case, his return is the result of a successful dynamic factor allocation strategy. The second possibility, however, is that because stocks are priced to yield a positive expected return, his excess return arose simply from having equity exposure (however erratic) over time. In that case, his return is ultimately a consequence of static factor allocation, regardless of his time-varying equity weights.

One could attempt to distinguish between these two possibilities in a number of ways. We propose a simple and straightforward method for evaluating the dynamic skill of our manager by noting the following identity:[3]

$$E\left(w_t R_t\right) = E\left(w_t\right) E\left(R_t\right) + \operatorname{cov}\left(w_t, R_t\right). \tag{2}$$

In Equation 2, the average portfolio return for our hypothetical manager, $E(w_t R_t)$, is decomposed into two parts. We define the first term on the right-hand side of the equation, $E(w_t)E(R_t)$, as the *static allocation effect*. This term captures the portion of the return gleaned from a static allocation to the equity market. Any weight on an asset that has a positive expected return can be expected to generate a positive return. In this particular example, our manager can be expected to generate a positive return, on average, simply by allocating a positive weight to the stock market.

We define the second term on the right-hand side of Equation 2, $\operatorname{cov}(w_t,R_t)$, as the *dynamic allocation effect*. This term captures the portion of our manager's return that is attributable to his ability to time the equity market. If our investor's portfolio weight in stocks is large when market returns are high and small when market returns are low, we would observe $\operatorname{cov}(w_t,R_t) > 0$. If we observe $\operatorname{cov}(w_t,R_t) = 0$, we would conclude that the manager's performance arises from a simple positive static exposure to the equity market and that he demonstrates no meaningful ability to allocate portfolio weights tactically. Note that if $\operatorname{cov}(w_t,R_t) < 0$, the manager may be actively destroying value. If our manager bought the market during periods of positive returns and shorted the market during periods of negative returns, his average weight in the market may be zero—$E(w_t) = 0$—and the value he adds may be characterized as arising entirely from his dynamic allocation skill. Traditional attribution analysis is designed only to identify and measure manager skill in factor allocation and does not distinguish between static and dynamic components.

In analyzing manager performance, distinguishing between static and dynamic exposures may be important for a number of reasons. Although static exposure to new risk factors is valuable, static exposure to known risk factors (the stock market in the previous example) may be easily replicable. One criticism of an alpha stream that is characterized by static factor exposures is that it can be replicated by low-cost allocations to passive indices in a buy-and-hold portfolio (although such replication may be approximately possible only for the limited number of factors for which passive funds exist, such as size and value). Our decomposition may help investors assess the portion of a manager's alpha that results from static allocations to known risk factors, particularly those that may be accessed passively through style indices. With that in mind, quantifying the dynamic component of a manager's alpha may represent an important rationale in justifying active management fees because a dynamic strategy is arguably less replicable than a static one.

Measuring a manager's static exposure is also useful in identifying persistent manager bias relative to a benchmark and in the construction of a "normal portfolio." Normal portfolios represent a manager's preferred allocation in the absence of views (see Black and Litterman 1992) and, among other things, can be used as a benchmark when no explicit benchmark exists. In such a setting, a candidate for a normal portfolio or benchmark may be one that admits little or no static allocation effect (i.e., the normal portfolio should closely match the average style of the managed portfolio).

Decomposing added value into static and dynamic components better characterizes managers' investment approaches, styles, and sources of added value and is thus useful to investors.

Methodology

Elegant and intuitive, the Brinson attribution methodology is an industry standard by any measure. Before we formally expand on it to introduce our framework, a review of the original univariate (single factor dimension) Brinson approach is warranted.

Review of Traditional Brinson Analysis. Over a single period, a manager's added value relative to a benchmark can be decomposed into allocation and security selection components as follows:

$$\begin{aligned}
&\sum_{i=1}^{N}\left(w_i^p R_i^p - w_i^b R_i^b\right) \\
&\quad = \text{Added value} \\
&\quad = \sum_{i=1}^{N}\left(w_i^p - w_i^b\right)\left(R_i^b - R^b\right) \\
&\qquad + \sum_{i=1}^{N} w_i^p \left(R_i^p - R_i^b\right) \\
&\quad = \text{Allocation effect} \\
&\qquad + \text{Security selection effect,}
\end{aligned} \tag{3}$$

where N is the number of factor groupings (i.e., 12 industry sectors or 10 price-to-book-value deciles); w_i^p, w_i^b, R_i^p, and R_i^b are the weights and returns for factor group i in both the manager portfolio and the benchmark; and R^b is the benchmark return. Note that the superscripts p and b refer to portfolio and benchmark, respectively. In the original Brinson analysis, *allocation effect* is a measure of a manager's skill at allocating among the industry sectors. More generally, we can measure a manager's skill at allocating among value, size, momentum, and other factor quintiles. *Security selection effect* is a measure of a manager's ability to overweight the higher-return stocks within these groups. Note that classical Brinson analysis also includes a term for the interaction between allocation and selection effects (see Brinson, Hood, and Beebower 1995). Our selection effect captures the sum of classical selection and interaction effects. Although important in some applications, the distinction is not critical here.[4]

Classical Brinson attribution was designed for single-period analysis that assumes static portfolio holdings; it does not directly allow for multiperiod analysis when portfolio weights are actively changed. Methodologies for multiperiod attribution analysis have been developed to better account for intertemporal decision making. One of the most commonly used techniques is to repeat the standard Brinson analysis over T periods and then take a simple average:

$$\begin{aligned}
&\frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}\left(w_{i,t}^p R_{i,t}^p - w_{i,t}^b R_{i,t}^b\right) \\
&\quad = \text{Avg. added value} \\
&\quad = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}\left(w_{i,t}^p - w_{i,t}^b\right)\left(R_{i,t}^b - R_t^b\right) \\
&\qquad + \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N} w_{i,t}^p \left(R_{i,t}^p - R_{i,t}^b\right) \\
&\quad = \text{Avg. allocation effect} \\
&\qquad + \text{Avg. security selection effect.}
\end{aligned} \tag{4}$$

The arithmetic added value is, naturally, different from geometric added value; modified methodologies exist that allow for geometric attribution.[5]

Weakness of Traditional Brinson Analysis. Multiperiod attribution yields a more complete picture of how an active manager generates alpha than does single-period attribution. Neither single-period nor multiperiod Brinson analysis, however, is able to characterize a manager's dynamic allocation, or market-timing, skill. To see this weakness, let us consider two portfolio managers who allocate between a value index and a growth index. Both managers are measured against a benchmark that is

50 percent invested in each index. The "static" manager, aware of the historical outperformance of value over growth, allocates a constant 80 percent of his portfolio weight to the value index and 20 percent to the growth index. The "dynamic" manager attempts to time the market and varies her portfolio weights on the basis of whether she believes growth will outperform value in a given period.

Suppose we observe the performance of these two managers over three years (**Table 1**). The static manager behaves predictably: At the beginning of each year, his portfolio weights are 80 percent value and 20 percent growth. The dynamic manager, however, places the majority of her portfolio weights in the growth index during Year 2 (when growth outperforms value). During Years 1 and 3 (when value outperforms growth), she places the majority of her portfolio weights in the value index. The results of these portfolio weights reveal that she is apparently successful in dynamically forecasting returns and responsively allocating portfolio weights.

How do the managers compare if we apply a classical multiperiod Brinson analysis to their portfolios? Using the arithmetic average approach, we can determine the average allocation and security selection effects as follows:

$$\begin{aligned}
&\text{3-yr. avg. added value}\\
&= \frac{1}{3}\sum_{t=1}^{3}\sum_{i=1}^{2}\left(w_{i,t}^{p}-w_{i,t}^{b}\right)\left(R_{i,t}^{b}-R_{t}^{b}\right)\\
&\quad+\frac{1}{3}\sum_{t=1}^{3}\sum_{i=1}^{2}w_{i,t}^{p}\left(R_{i,t}^{p}-R_{i,t}^{b}\right) \qquad (5)\\
&= \text{Avg. allocation effect}\\
&\quad + \text{Avg. security selection effect.}
\end{aligned}$$

In this example, by construction, the average security selection effect is zero for both managers: They are allocating to indices and make no stock selection decisions. Therefore, the only source of their added value is factor allocation—their ability to allocate between value and growth indices. Here, a classical multiperiod Brinson analysis says exactly the same thing for *both* managers: The value added from the factor allocation of both managers is the same—1.5 percent. Clearly, the investment strategies of these two managers are markedly different, but classical Brinson methodology fails to distinguish between them.

Table 1. Static and Dynamic Manager Performance

	Year 1	Year 2	Year 3	Average
Sector				
Value return	16%	8%	3%	9.00%
Growth return	4	10	−2	4.00
Equal-weighted benchmark				
Value weight	50%	50%	50%	50.00%
Growth weight	50	50	50	50.00
Benchmark return	10.0	9.0	0.5	6.50
Static manager				
Value weight	80%	80%	80%	80.00%
Growth weight	20	20	20	20.00
Return	13.6	8.4	2.0	8.00
Added value	3.6	–0.6	1.5	1.50
Static added value				1.50
Dynamic added value				0.00
Dynamic manager				
Value weight	75%	10%	64%	49.67%
Growth weight	25	90	36	50.33
Return	13.0	9.8	1.2	8.00
Added value	3.0	0.8	0.7	1.50
Static added value				–0.02
Dynamic added value				1.52

A New Framework for Capturing Manager Dynamic Factor Allocation Ability. We propose a straightforward way to divide existing allocation effect into static and dynamic components. As in Equation 2, for each factor i considered, we separate the allocation effect into static and dynamic components by using the following identity:

$$
\begin{aligned}
&E\left[\left(w_{i,t}^{p}-w_{i,t}^{b}\right)\left(R_{i,t}^{b}-R_{t}^{b}\right)\right] \\
&= E\left(w_{i,t}^{p}-w_{i,t}^{b}\right)E\left(R_{i,t}^{b}-R_{t}^{b}\right) \\
&\quad + \operatorname{cov}\left[\left(w_{i,t}^{p}-w_{i,t}^{b}\right),\left(R_{i,t}^{b}-R_{t}^{b}\right)\right] \qquad (6)\\
&= \text{Static allocation effect} \\
&\quad + \text{Dynamic allocation effect.}
\end{aligned}
$$

The left-hand side of Equation 6 is simply the average allocation effect from the standard attribution model, and the right-hand side is the decomposition into the static and dynamic components. In a univariate analysis, in which a single factor is categorized into N groups and studied over T periods, Equation 6 can be summed for each factor in order to decompose the factor selection into dynamic and static components for the entire portfolio:

$$
\begin{aligned}
&\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(w_{i,t}^{p}-w_{i,t}^{b}\right)\left(R_{i,t}^{b}-R_{t}^{b}\right) \\
&= \sum_{i=1}^{N}\left[\frac{1}{T}\sum_{t=1}^{T}\left(w_{i,t}^{p}-w_{i,t}^{b}\right)\right]\left[\frac{1}{T}\sum_{t=1}^{T}\left(R_{i,t}^{b}-R_{t}^{b}\right)\right] \\
&\quad + \sum_{i=1}^{N}\left\{\frac{1}{T}\sum_{t=1}^{T}\left[\left(w_{i,t}^{p}-w_{i,t}^{b}\right)-\frac{1}{T}\sum_{j=1}^{T}\left(w_{i,j}^{p}-w_{i,j}^{b}\right)\right]\right. \qquad (7)\\
&\quad \left.\left[\left(R_{i,t}^{b}-R_{t}^{b}\right)-\frac{1}{T}\sum_{j=1}^{T}\left(R_{i,j}^{b}-R_{j}^{b}\right)\right]\right\} \\
&= \text{Static allocation} + \text{Dynamic allocation.}
\end{aligned}
$$

Although straightforward, the calculation of the covariance term is somewhat cumbersome and can be accomplished more easily by taking the difference between the total allocation added value and static added value. Thus, the total allocation added value (including the decomposition into static and dynamic components) and security selection added value for the portfolio can be computed as follows:[6]

$$
\begin{aligned}
&\text{Allocation added value} \\
&= \frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(w_{i,t}^{p}-w_{i,t}^{b}\right)\left(R_{i,t}^{b}-R_{t}^{b}\right); \qquad (8a)
\end{aligned}
$$

$$
\begin{aligned}
&\text{Static allocation added value} \\
&= \sum_{i=1}^{N}\left[\frac{1}{T}\sum_{t=1}^{T}\left(w_{i,t}^{p}-w_{i,t}^{b}\right)\right]\left[\frac{1}{T}\sum_{t=1}^{T}\left(R_{i,t}^{b}-R_{t}^{b}\right)\right]; \qquad (8b)
\end{aligned}
$$

$$
\begin{aligned}
&\text{Dynamic allocation added value} \\
&\quad = \text{Allocation added value} \qquad (8c)\\
&\qquad - \text{Static added value;}
\end{aligned}
$$

$$
\begin{aligned}
&\text{Security selection added value} \\
&= \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}w_{i,t}^{p}\left(R_{i,t}^{p}-R_{i,t}^{b}\right). \qquad (8d)
\end{aligned}
$$

When we use this approach, our evaluation of the two fund managers is quite different. The bottom of Table 1 shows the decomposed allocation effect based on Equation 8. The static manager's performance is precisely as we would expect; all 1.5 percent of the static manager's added value is attributed to his static factor exposure. The dynamic attribution is zero for this manager. For the dynamic manager, the static added value is roughly zero, with the dynamic strategy driving the full 1.5 percent of added value.

The value added from security selection may be similarly decomposed into static and dynamic components. Recall that our "security selection" is actually the sum of an interaction term and the classical security selection. This difference raises interpretation issues with respect to decomposing security selection into static and dynamic components—issues that we do not address in this article.

Finally, we emphasize that, similar to the traditional Brinson analysis, this methodology is performed strictly *ex post*. For example, the expression for static added value (Equation 8b) requires average portfolio weights at returns, which cannot be known until the end of the period being examined. Thus, the attribution model does not reflect any real investment process but rather seeks to characterize the effect of investment decisions *ex post*. This methodology can be used *ex ante*, however, in the development of quantitative strategies (which typically require backtesting) to characterize historical sources of added value more fully. This analysis may aid in the selection of quantitative strategies for implementation.

An Application

To illustrate this dynamic attribution framework further, we applied our analysis to the equity holdings of several large and well-known mutual funds, whose reported objectives as of June 2009 are summarized in **Exhibit 1**. The purpose of this exercise was not to present a comprehensive examination of all active managers who use the dynamic attribution model adduced in our study. Rather, our goal was to (1) illustrate the application of the proposed methodology, (2) interpret and analyze results of the methodology, and (3) demonstrate the robustness of our approach to small variations in model specifications.

Exhibit 1. Fund Objectives, June 2009

Mutual Fund	Investment Objective
American Funds Fundamental Investors	Seeking to provide long-term growth of capital and income, primarily through investments in common stocks.
American Funds Growth Fund of America	Seeking to provide long-term growth of capital through a diversified portfolio of common stocks. Has the flexibility to invest wherever the best growth opportunities may be. The fund emphasizes companies that appear to offer opportunities for long-term growth and may invest in cyclical companies, turnarounds, and value situations.
Dodge & Cox Stock Fund	Seeking long-term growth of principal and income. The fund invests primarily in a broadly diversified portfolio of common stocks. In selecting investments, the fund invests in companies that, in Dodge & Cox's opinion, appear to be temporarily undervalued by the stock market but have a favorable outlook for long-term growth.
Fidelity Advisor Small Cap Fund	Investing at least 80 percent of assets in securities of companies with small market capitalizations (companies with market capitalizations similar to those of companies in the Russell 2000 Index or the S&P SmallCap 600). Investing in either growth stocks or value stocks or both. Normally invests primarily in common stocks.
Fidelity Contrafund	Investing in securities of companies whose value Fidelity Management and Research (FMR) believes is not fully recognized by the public. Investing in either growth stocks or value stocks or both.
Fidelity Magellan Fund	Investing primarily in common stocks. Investing in either growth stocks or value stocks or both.
Janus Fund	Investing primarily in common stocks selected for their growth potential. Although the fund may invest in companies of any size, it generally invests in larger, more established companies. The portfolio manager applies a bottom-up approach in choosing investments.
T. Rowe Price Small-Cap Stock Fund	Investing at least 80 percent of net assets in stocks of small companies. Security selection may reflect either a growth or a value investment approach.
Vanguard 500 Index Fund Investor Shares	Investing in stocks in the S&P 500 Index, representing 500 of the largest U.S. companies. Goal is to closely track the index's return, which is considered a gauge of overall U.S. stock returns.
Wells Fargo Advantage Small Cap Value Fund	Seeking capital growth by investing primarily in the undervalued stock of small-cap companies.

Sources: American Funds, Dodge & Cox, FMR, Janus, T. Rowe Price, Vanguard, Wells Fargo.

In our application, we examined large-cap and small-cap managers. For the large-cap managers, we required the fund to have been live in 1980; for the small-cap managers, we required the fund to have been live in 2000. Each fund also had to be ranked within the top 100 funds by AUM in its category in 2008. From the funds meeting these criteria, we selected six large-cap and three small-cap funds, with the understanding that the resulting sample had significant survivorship bias. We included in our sample an index fund (the Vanguard 500 Index Fund Investor Shares) to illustrate the baseline case.

We obtained information about holdings for each of these mutual funds through 2008 from the Thomson Reuters mutual fund database, which uses quarterly U.S. SEC filings. We obtained return data from CRSP. To benchmark the large-cap funds, we created a "Cap 1000" benchmark. The constituents of this benchmark were the largest 1,000 U.S.-listed companies (by market capitalization) at the beginning of each calendar year. This methodology produced an index analogous to the Russell 1000 Index. Similarly, for the small-cap funds, we created a "Cap 2000" benchmark whose constituents were the next largest 2,000 U.S.-listed companies at the beginning of each calendar year and that was roughly analogous to the Russell 2000 Index. For the Vanguard 500 Index Fund, which seeks to mimic the S&P 500 Index, we created a "Cap 500" index as a benchmark.

To demonstrate the type of output this analysis generates, we applied our dynamic allocation attribution to each of the funds in our sample. Active managers use multiple strategies along multiple factor dimensions. For ease of illustration, however, we limited our initial analysis to three factors common in the investment industry and considered them separately: industry sectors, value/growth styles, and small-cap/large-cap styles. Of course, managers may choose to base their holdings on other factors, such as momentum, volatility, and market beta; our methodology can be easily extended to account for these additional factors.

For the industry sectors, holdings for both the mutual fund and its benchmark were assigned to subportfolios on the basis of the Fama–French 12 industry classifications. For the value/growth style groups based on book-to-market ratios, we assigned holdings to 1 of the 10 book-to-market subportfolios on the basis of NYSE breakpoints at the beginning of each calendar year.[7] For the small-cap/large-cap groups based on market capitalization, we assigned holdings to 1 of the 10 capitalization subportfolios on the basis of NYSE breakpoints at the beginning of each calendar year.

Although most of these funds invest primarily in U.S. equity, they typically also have cash holdings, some foreign stocks, and perhaps even some fixed-income investments. Therefore, the added-value numbers reflect an analysis of the performance based only on the funds' estimated U.S. equity holdings.

The results of the three univariate analyses based on industry sectors and value/growth and small-cap/large-cap deciles are reported in **Table 2**. The first three columns contain the name of the fund, the starting date (as recorded by the Thomson Reuters database), and the benchmark (our construction). The fourth, fifth, and sixth columns contain the total allocation effect and the decomposition into static and dynamic allocation effects. The last two columns contain the security selection and total added value of each fund relative to its benchmark. We used quarterly data in the analysis and annualized the added-value numbers.[8]

Security selection makes up the majority of the overall added value in all three panels. At first glance, this result appears to be extremely positive news for the handful of active managers that we selected for our study. Consistent with the stated objectives of the funds, the managers did not pursue a strategy of systematically tilting toward growth or value stocks or particular industry sectors, although there appears to be a systematic bias for large-cap stocks, which hurt long-term performance (we will explore this effect later in the article). For this sample, timing skill appears to be strongest in industry sector allocation; an average added value of 96 bps came from sector timing. Dynamic allocation associated with value and growth is weak; on average, managers were able to generate 26 bps from value- and growth-style timing. Dynamic allocation to small-cap and large-cap stocks is actually negative, on average.

The static allocations to industry sectors or value/growth style do not appear to have contributed to added value. The static allocation to size, however, contributed significantly to *negative* performance; this result is driven by an average overweight to large-cap stocks. Because all the funds have significant AUM, they may be forced to shy away from the small-cap names in their benchmarks for reasons of liquidity and capacity. The outcome would be a systematic large-cap bias in these portfolios, resulting in negative static allocation alphas over time.

As stated in the funds' objectives, security selection is the dominant investment performance driver. Note, however, that the results in Table 2 correspond to three separate analyses that examine only one factor at a time. Allocation to factors that are orthogonal to the one being considered will show up as security selection, so the high security selection added value may be the result of managers allocating to factors that were not considered in the univariate analysis. For example, the analysis for industry groups (Panel A) did not explicitly consider the effect of the value/growth factor tilt in the portfolio over time. To the extent that the value factor is orthogonal to industry effects, allocations to it would be absorbed by security selection in Panel A.

One way to address this issue is to consider multiple factors simultaneously. This method can be carried out by creating mutually exclusive subportfolios that are based on multiple sorts along several risk dimensions. To illustrate this approach, we created factors that are based on the Fama–French 12 industry groups, five NYSE book-to-market groups, and five market-cap groups. This three-way sort resulted in 300 subportfolios and allowed us to consider a manager who is simultaneously allocating to these three factors. **Table 3** presents the results of this analysis.

The allocation effect summarizes the total value added from the manager's static allocation to industry sectors, value tilt, and size tilt, as well as the dynamic allocation effect. In this case, the majority of added value is still attributable to security selection (4 percent, on average), which suggests that managers are good stock pickers and/or are allocating to factors that we are not considering explicitly. In this limited sample, approximately 1 percent of the added value is attributable to dynamic allocation skill.

Another feature that Table 3 illustrates is the robustness of our approach. Generally, the nature of holdings-based analysis is such that the portion of added value attributable to the allocation effect increases with the number of groups into which

Table 2. Univariate Manager Dynamic Allocation Skill Attribution through December 2008

Fund	Start Date	Benchmark	Allocation Effect	Static	Dynamic	Security Selection	Total
A. Fama–French 12 industry groups							
Vanguard 500 Index Fund Investor Shares	9/1980	Cap 500	0.21%	0.06%	0.15%	0.08%	0.29%
American Funds Fundamental Investors	3/1980	Cap 1000	0.32	–0.25	0.57	2.32	2.64
American Funds Growth Fund of America	3/1980	Cap 1000	0.88	–0.33	1.21	3.39	4.27
Dodge & Cox Stock Fund	3/1980	Cap 1000	0.57	–0.17	0.74	2.28	2.85
Fidelity Contrafund	12/1980	Cap 1000	0.76	–0.26	1.02	2.84	3.61
Fidelity Magellan Fund	6/1981	Cap 1000	0.70	–0.22	0.92	1.74	2.44
Janus Fund	3/1980	Cap 1000	1.02	–0.17	1.19	3.78	4.80
Fidelity Advisor Small Cap Fund	9/1999	Cap 2000	0.19	–0.44	0.63	5.13	5.32
T. Rowe Price Small-Cap Stock Fund	6/1993	Cap 2000	0.79	–0.16	0.95	1.61	2.40
Wells Fargo Advantage Small Cap Value Fund	6/1998	Cap 2000	2.30	0.90	1.40	6.53	8.83
Active fund average			0.84	–0.12	0.96	3.29	4.13
B. Book-to-market groups							
Vanguard 500 Index Fund Investor Shares	9/1980	Cap 500	0.08%	0.01%	0.07%	0.21%	0.29%
American Funds Fundamental Investors	3/1980	Cap 1000	0.44	0.17	0.27	2.16	2.60
American Funds Growth Fund of America	3/1980	Cap 1000	–0.09	–0.17	0.08	4.38	4.30
Dodge & Cox Stock Fund	3/1980	Cap 1000	0.84	0.36	0.48	2.00	2.84
Fidelity Contrafund	12/1980	Cap 1000	0.45	0.12	0.33	3.23	3.68
Fidelity Magellan Fund	6/1981	Cap 1000	0.22	–0.03	0.24	2.21	2.43
Janus Fund	3/1980	Cap 1000	0.37	–0.24	0.62	4.45	4.82
Fidelity Advisor Small Cap Fund	9/1999	Cap 2000	–0.63	–0.06	–0.57	5.68	5.05
T. Rowe Price Small-Cap Stock Fund	6/1993	Cap 2000	0.70	0.02	0.68	1.61	2.32
Wells Fargo Advantage Small Cap Value Fund	6/1998	Cap 2000	0.60	0.41	0.19	7.79	8.39
Active fund average			0.32	0.06	0.26	3.73	4.05
C. Capitalization groups							
Vanguard 500 Index Fund Investor Shares	9/1980	Cap 500	–0.24%	–0.25%	0.01%	0.53%	0.29%
American Funds Fundamental Investors	3/1980	Cap 1000	–0.15	–0.05	–0.10	2.77	2.61
American Funds Growth Fund of America	3/1980	Cap 1000	–1.17	–0.89	–0.28	5.48	4.31
Dodge & Cox Stock Fund	3/1980	Cap 1000	0.38	–0.01	0.39	2.47	2.85
Fidelity Contrafund	12/1980	Cap 1000	–2.32	–1.62	–0.70	5.96	3.64
Fidelity Magellan Fund	6/1981	Cap 1000	–1.83	–1.22	–0.61	4.26	2.43
Janus Fund	3/1980	Cap 1000	–0.97	–0.96	–0.01	5.78	4.82
Fidelity Advisor Small Cap Fund	9/1999	Cap 2000	–0.06	–1.07	1.01	5.33	5.27
T. Rowe Price Small-Cap Stock Fund	6/1993	Cap 2000	–2.45	–1.30	–1.16	5.10	2.65
Wells Fargo Advantage Small Cap Value Fund	6/1998	Cap 2000	–1.80	–1.13	–0.67	10.60	8.81
Active fund average			–1.15	–0.92	–0.24	5.31	4.15

the portfolio is subdivided. This effect is easy to comprehend. If we were to take the number of groups to the extreme—at which point each security would be an individual group—all the added value would be, by definition, attributable to the allocation effect. In our current application, we can see that even with 300 groups, our results are very robust, with security selection still receiving the lion's share of added value.

The managers of the funds in our sample—who experienced tremendous success over the long run (the selection criteria required large AUM and a long track record)—appear to demonstrate significant skill in security selection and in timing industry

Table 3. Multivariate Manager Dynamic Allocation Skill Attribution through December 2008

Fund	Start Date	Benchmark	Allocation Effect	Static	Dynamic	Security Selection	Total
Vanguard 500 Index Fund Investor Shares	9/1980	Cap 500	–0.05%	–0.06%	0.01%	0.35%	0.29%
American Funds Fundamental Investors	3/1980	Cap 1000	0.45	–0.15	0.60	2.14	2.59
American Funds Growth Fund of America	3/1980	Cap 1000	–0.22	–1.07	0.85	4.54	4.32
Dodge & Cox Stock Fund	3/1980	Cap 1000	1.27	–0.49	1.75	1.57	2.84
Fidelity Contrafund	12/1980	Cap 1000	–1.05	–1.94	0.89	4.74	3.69
Fidelity Magellan Fund	6/1981	Cap 1000	–0.96	–1.41	0.45	3.39	2.43
Janus Fund	3/1980	Cap 1000	0.20	–1.15	1.34	4.64	4.84
Fidelity Advisor Small Cap Fund	9/1999	Cap 2000	0.92	–1.34	2.27	4.00	4.93
T. Rowe Price Small-Cap Stock Fund	6/1993	Cap 2000	–1.50	–1.54	0.04	3.77	2.27
Wells Fargo Advantage Small Cap Value Fund	6/1998	Cap 2000	0.32	–0.25	0.56	8.04	8.35
Active fund average			–0.06	–1.04	0.97	4.09	4.03

Note: Subportfolios comprise the Fama–French 12 industry groups, five book-to-market groups, and five market-cap groups.

sectors in their fund management; they also show modest skill in timing growth- and value-style selection. They show negative static allocation skill, however, in selecting between small-cap and large-cap stocks, although this failure may be driven by liquidity and capacity issues associated with the small-cap names in the benchmarks. Surprisingly, value investing, which has been documented as one of the more consistent outperforming strategies, does not play a role in these funds.

In the cross section of managers in our sample, we found a wide disparity in sources of added value, as well as in the split between dynamic and static allocation approaches. We refrain from drawing additional conclusions from our attribution. The managers that we selected for our sample may be either positive outliers or truly high-alpha managers. We have not performed the proper econometrics to distinguish the two hypotheses. Our attribution is intended simply to illustrate our methodology.

Conclusion

We have proposed a dynamic allocation attribution methodology that retains the intuition and familiar characteristics of traditional Brinson attribution analysis. In addition to distinguishing between security selection and factor selection, our methodology subdivides the allocation effect into static and dynamic components. The static component measures the performance attributable to the persistent factor profile of the manager's portfolio. The dynamic component measures the performance attributable to the manager's timing ability. We believe that distinguishing between static and dynamic allocation skills in the factor domain is important because doing so provides further insight into the investment approach of managers and more fully characterizes drivers of manager alpha.

This article qualifies for 1 CE credit.

Notes

1. See Investment Company Institute, *2009 Investment Company Fact Book* (Washington, DC: Investment Company Institute, 2009).
2. See, for example, Carino (1999); Laker (2005); Menchero (2000, 2004); and Davies and Laker (2001).
3. Grinblatt and Titman (1993) made a similar observation in measuring manager performance without benchmarks.
4. The interaction effect is positive when a manager overweights sectors in which she has a positive stock selection ability and underweights sectors in which she does not. The effect is often added to classical security selection to simplify the analysis (see Fabozzi and Markowitz 2002).
5. For convenience, our focus is on arithmetic attribution analysis. For representative examples of a geometric approach, see Bacon (2002) and Menchero (2000/2001).
6. Note that when summing allocation effects among all factors, $\sum_{i=1}^{N}\left(w_i^p - w_i^b\right)R^b = 0.$ Thus, the portfolio-level returns (R_t^b) can be omitted from the allocation effects in Equations 8a–8c without changing the aggregate result, which eases the complexity of calculating allocation effects at the portfolio level.
7. We defined book-to-market ratio as the ratio of a company's book equity to its market equity. We calculated the book value of equity as the book value of stockholders' equity plus balance sheet deferred taxes and investment tax credit minus the book value of preferred stock (as of the most recent reporting date in Capital IQ Compustat). We calculated the market value of equity as the share price multiplied by the number of shares outstanding.
8. Total added value may be slightly different for each fund among the panels. This difference arises from the fact that we were unable to obtain industry or book-to-market information for every stock in the sample.

References

Bacon, Carl. 2002. "Excess Returns—Arithmetic or Geometric?" *Journal of Performance Measurement*, vol. 6, no. 3 (Spring):23–31.

Black, Fischer, and Robert Litterman. 1992. "Global Portfolio Optimization." *Financial Analysts Journal*, vol. 48, no. 5 (September/October):28–43.

Brinson, Gary P., and Nimrod Fachler. 1985. "Measuring Non-U.S. Equity Portfolio Performance." *Journal of Portfolio Management*, vol. 11, no. 3 (Spring):73–76.

Brinson, Gary P., L. Randolph Hood, and Gilbert L. Beebower. 1995. "Determinants of Portfolio Performance." *Financial Analysts Journal*, vol. 51, no. 1 (January/February):133–138.

Carino, David. 1999. "Combining Attribution Effects over Time." *Journal of Performance Measurement*, vol. 3, no. 4 (Summer):5–14.

Daniel, Kent, Mark Grinblatt, Sheridan Titman, and Russ Wermers. 1997. "Measuring Mutual Fund Performance with Characteristic-Based Benchmarks." *Journal of Finance*, vol. 52, no. 3 (July):1035–1058.

Davies, Owen, and Damien Laker. 2001. "Multiple-Period Performance Attribution Using the Brinson Model." *Journal of Performance Measurement*, vol. 6, no. 1 (Fall):12–22.

Fabozzi, Frank J., and Harry M. Markowitz. 2002. *The Theory and Practice of Investment Management*. New York: John Wiley & Sons.

Fama, Eugene F., and Kenneth R. French. 1992. "The Cross-Section of Expected Stock Returns." *Journal of Finance*, vol. 47, no. 2 (June):427–465.

———. 1993. "Common Risk Factors in the Returns on Stocks and Bonds." *Journal of Financial Economics*, vol. 33, no. 1 (February):3–56.

Grinblatt, Mark, and Sheridan Titman. 1993. "Performance Measurement without Benchmarks: An Examination of Mutual Fund Returns." *Journal of Business*, vol. 66, no. 1 (January):47–68.

Laker, Damien. 2005. "Toward Consensus on Multiple-Period Arithmetic Attribution." *Journal of Performance Measurement*, vol. 9, no. 3 (Spring):26–37.

Menchero, Jose G. 2000. "An Optimized Approach to Linking Attribution Effects over Time." *Journal of Performance Measurement*, vol. 5, no. 1 (Fall):36–42.

———. 2000/2001. "A Fully Geometric Approach to Performance Attribution." *Journal of Performance Measurement*, vol. 5, no. 2 (Winter):22–30.

———. 2004. "Multiperiod Arithmetic Attribution." *Financial Analysts Journal*, vol. 60, no. 4 (July/August):76–91.

