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Michael implements strategies developed by Research Affiliates. Previously, he served as a managing director at the University of Virginia Investment Management Company, working on portfolio and risk management. He also has worked for Sunsuper, an Australian superannuation fund, and at UBS Global Asset Management across four continents. Michael received his MS in financial mathematics from the University of Chicago, and a BA with honors in applied mathematics from the University of Sydney. He recently completed a Masters in Statistics from the University of Virginia. He is a member of the CFA Institute and of the CFA Society of Washington DC.

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Large-scale index-based investing doesn't just reflect equity market values; it affects them, too. Implementing an index fund results in direct and indirect costs. The direct costs are commissions and transaction-related fees; the indirect costs include, among other things, the market price impact of trading in size. In this paper, we describe an approach to analyzing indirect trade costs incurred in rebalancing, and we present our findings related to cap-weighted, equal-weighted, and fundamentals-weighted indices.

OUR POINT OF VIEW

Academics and practitioners have written a great deal about equity trade cost analysis.¹ Our work was broadly guided by the theoretical framework Robert Almgren, Chee Thum, Emmanuel Hauptmann, and Hong Li set forth in their 2005 paper, "Direct Estimation of Equity Market Impact."² Following their lead, we focused on the price effect of the trader's own actions, and we incorporated factors such as average daily volume and turnover into our analysis. However, the model we developed is considerably more parsimonious than the one described by Almgren's research team.

In principle, the market impact cost of an index-based investment strategy can be decomposed into two components: an imputed reduction in the performance of the underlying index, and an observed difference between the performance of the fund and that of the index.³ The relative contribution of the two components will depend in part on the trading schedule. For example, if an investor replicates the index by trading "market on close," then there is no performance gap between the index and fund, and the reduced performance of the underlying index accounts for the entire market impact cost. Concurrent trading in the same stocks by other market participants—including other managers tracking the same index—may also affect the variance between fund and index results.

This paper centers on market impact cost at the index level. Unlike a fund's performance versus an index, this component cannot be measured directly; it is not possible to observe how a fund would have performed had it not been funded. However, statistical techniques can be used to analyze a large number of index changes and calibrate a theoretical index-level model.

A SIMPLIFIED MARKET IMPACT MODEL

The Research Affiliates study used an internal database containing fundamental index rebalances from 2009–2013. The aggregate trades that would be required to replicate these rebalances amount to 1–40% of the stocks' average daily volumes (ADVs).

From the empirical data, we determined that a linear market impact model represented a good trade-off between simplicity and explanatory power:

$$\frac{\Delta p}{p} = k \frac{X}{V}$$

Here, p is the pre-trade price of a stock, k is a constant that may depend on the individual market, X is the number of shares bought or sold, and V is the aggregate ADV across all the company's share classes. Δp is the price change, that is, the difference between the stock's post-trade and pre-trade prices.

The Appendix to this paper details the mathematics of applying the linear trade impact model.

THE FACTORS THAT DETERMINE THE RETURN IMPACT OF INDEX TRADING

As the Appendix demonstrates, the market impact of a rebalance on the performance of an index is the product of four factors: base impact, index breadth, effective turnover, and index tilt.

The *base impact* factor is the ratio of the assets under management in a given strategy to the dollar value of shares traded daily across all the stocks in the universe of interest, scaled by a constant factor. The key observation is that, under our assumptions, the cost is linear in strategy size.

The inverse of the *index breadth* factor (1/index breadth) is the fraction of the total universe trade volume captured by the index. The index breadth is 1 for a portfolio that contains every stock in the initial universe; it can fall by an order of magnitude or more for a small-stock or a highly concentrated index.

The *effective turnover* factor reflects the fact that a simple annual rebalancing can have quite different market impact consequences depending on the structural relationship between stocks replaced and stocks retained. Effective turnover is proportional to the squared turnover if the portfolio doesn't change constituents at rebalance and to the turnover itself if the portfolio replaces all its constituents.

Finally, the *index tilt* indicates how far the index departs from the cheapest to trade, a volume-weighted index. Index tilt is defined as the weighted average of index weights relative to a volume-weighted index. The volume-weighted portfolio has tilt of 1, and any other portfolio has tilt above 1.

THE MARKET IMPACT OF ANNUAL REBALANCING

Using this framework, we will compare the market impact cost of nine portfolios: Cap 1000, Equal-Weight 1000 (with the same constituents as the Cap 1000), and RAFI® 1000, each in three regions: U.S., Developed ex US, and Emerging. We will initially assume identical strategy size.

The index breadth is nearly identical, since it is about 1.0 for a broad portfolio. Therefore, any difference in the market impact cost between the portfolios will be due to the base impact, effective turnover, and index tilt.

With the same strategy size, the base impact is a function of the total trading volume of the universe. **Table 1** shows the snapshot at the end of June 2013.

The effective turnover is comprised of two components: a linear function for the additions and deletions and a quadratic function for the reweighting of existing securities. **Table 2** displays the component values that emerged from our study.

TABLE 1. BASE IMPACT SNAPSHOT AS OF JUNE 30, 2013		
REGION	VOLUME, \$B	VOLUME, % OF GLOBAL
U.S.	136	52%
Developed ex U.S.	96	36%
EM	31	12%
Global	264	100%

Source: Research Affiliates.

Because the fundamental size is more stable over time than the market capitalization, the turnover from additions and deletions is smaller for the fundamental indices compared to cap-weighted indices. On the other hand, the reweighting turnover is higher for the fundamental indices, since they require rebalancing against the price movement. The equal-weighted indices have the highest effective turnover, due to the very high turnover from additions and deletions (those occur at larger weights in an equal-weighted index).

In virtually all cases, both turnover components are much higher in the emerging markets than in the developed regions; this is due in part to the higher idiosyncratic volatility in the emerging markets.

Table 3 summarizes the index tilt for the portfolios.

Again, we note that the tilt increases dramatically as we move from the developed to the emerging markets region.

Finally, we can compare the aggregate measure of market impact (i.e., the product of the effective turnover, tilt, and inverse universe volume [**Table 4**]).

These numbers are scaled relative to the U.S. Cap 1000 portfolio. For example, the model predicts that, *at the same asset size*, the market impact cost of rebalancing a broad RAFI U.S. index is almost three times greater than that of a broad U.S. cap-weighted index.

Of course, the assets tracking cap-weighted indices (about \$7 trillion by P&I estimates)⁴ are much greater than the fundamentals-weighted index assets (approximately \$100B). Adjusted for that, it would seem that cap-weighted index investing creates 25 times larger market impact than fundamentals-weighted indexing. In reality, as the size of the cap-weighted strategies grew, new

TABLE 2. COMPONENTS OF EFFECTIVE TURNOVER

Index	Component	U.S.	Dev ex U.S.	EM
Cap 1000	T _{adds & deletes}	3.9%	6.6%	11.7%
	T _{reweightings}	5.2%	5.7%	9.2%
	Effective turnover	4.2%	6.9%	12.4%
RAFI 1000	T _{adds & deletes}	3.1%	4.4%	8.4%
	T _{reweightings}	19.9%	22.3%	33.8%
	Effective turnover	9.2%	12.0%	25.2%
EW 1000	T _{adds & deletes}	17.4%	18.9%	27.2%
	T _{reweightings}	19.9%	19.6%	26.4%
	Effective turnover	22.9%	24.2%	36.2%

Source: Research Affiliates.

TABLE 3. INDEX TILT

INDEX	U.S.	DEV EX U.S.	EM
Cap 1000	1.35	1.36	2.01
RAFI 1000	1.66	1.39	1.71
EW 1000	3.20	3.27	6.52

Source: Research Affiliates.

TABLE 4. MARKET IMPACT MEASURES

INDEX	U.S.	DEV EX U.S.	EM
Cap 1000	1.00	2.36	19.14
RAFI 1000	2.68	4.19	33.01
EW 1000	12.89	19.92	180.85

Source: Research Affiliates.

developments in the market acted to reduce the market impact cost. For example, index providers started to provide more lead time in pre-announcing index changes, thus encouraging liquidity providers to come in. Furthermore, the index portfolio managers become aware of the market impact and trade less aggressively. These and similar factors are not accounted for in the market impact model we presented, but are subjects for future research.

APPENDIX

Consider an annually rebalanced investment strategy implemented by trading to the precise target weights on the day of the rebalance. The market impact cost incurred from trading a single security s equals $k \frac{(A\Delta w_{p,s})^2}{V_s}$, where $w_{p,s}$ is its weight in the *post-rebalance*⁵ portfolio p (pre-rebalance weights are used for divested stocks); V_s is its ADV; and A is the amount of assets invested in the strategy. The reduction in performance due to the market impact of rebalancing is then

$$c = \frac{1}{A} \sum_s k \frac{(A\Delta w_{p,s})^2}{V_s} = k \frac{A}{V_u} \frac{V_u}{V_p} \sum_s \frac{\Delta w_{p,s}^2}{v_{p,s}} \quad (1)$$

Here V_u , V_p are the total ADV of the securities in the universe of interest⁶ and in the portfolio, respectively; and $v_{p,s} = \frac{V_s}{V_p}$ is the weight stock s would have if the portfolio were volume-weighted.

Let us denote the absolute fraction of the stock traded $\delta_{p,s} = \frac{|\Delta w_{p,s}|}{w_{p,s}}$ and assume for simplicity that $\delta_{p,s}$ ² is uncorrelated with the $\frac{w_{p,s}}{v_{p,s}}$. We can then write:

$$c = k \frac{A}{V_u} \frac{V_u}{V_p} \sum_s w_{p,s} \delta_{p,s}^2 \frac{w_{p,s}}{v_{p,s}} = k \frac{A}{V_u} \frac{V_u}{V_p} \sum_s w_{p,s} \delta_{p,s}^2 \frac{\sum_s w_{p,s} \frac{w_{p,s}}{v_{p,s}}}{\sum_s w_{p,s}} \quad (2)$$

As a step toward making the turnover structure more intuitive, let us denote the fraction of stocks added to the portfolio as $f_a = \sum_{added} w_{p,s}$; the fraction of stocks deleted as $f_d = \sum_{deleted} w_{p,s}$; and the weighted average fractional weight change of the stocks that are retained as $\bar{\delta} = \frac{\sum_{retained} w_{p,s} |\delta_{p,s}|}{\sum_{retained} w_{p,s}}$.

The two-way turnover of the portfolio is $T = f_a + f_d + (1 - f_a)\bar{\delta} = T_{a\&d} + T_r$ where $T_{a\&d}$ is the turnover from additions and deletions; T_r is the turnover from the retained stocks; and $\bar{\delta} = \frac{T_r}{1 - f_a}$.

Furthermore, let $\psi = \frac{\sum_{s \in r} w_{p,s} \delta_{p,s}^2}{(\sum_{s \in r} w_{p,s} \delta_{p,s})^2}$, the ratio of the average squared $\delta_{p,s}$ to the square of the average absolute value of $\delta_{p,s}$. The characteristic ψ depends on the distribution of rebalance trades. For example, if all trades were the same fraction of the traded security's weight, then $\psi = 1$; if the target weights were the same year to year, and the trades were driven exclusively by price drift,⁷ then $\psi \approx \frac{\pi}{2}$. We can then rewrite:

$$c = k \frac{A}{V_u} \frac{V_u}{V_p} (T + \psi(1 - f_a)T_r^2) \frac{\sum_s w_{p,s} \frac{w_{p,s}}{v_{p,s}}}{\sum_s w_{p,s}} \quad (3)$$

We can therefore identify the following factors that determine the return impact:

1. *base impact*, which only depends on the ratio of the strategy size to the total trading volume of the relevant universe;
2. $1/\text{index breadth}$, the fraction of the total universe volume captured by the portfolio;
3. *effective turnover*, which has a linear term for replacements and a quadratic term for retained stocks; and
4. *index tilt vs. volume benchmark*, defined as the weighted average of portfolio overweights relative to a volume-weighted portfolio.

The base impact reflects the cost of buying into the broad volume-weighted portfolio of size A . The critical observation is that the cost is linear in strategy size under our assumptions.

The index breadth is 1 for a portfolio that contains every stock in the initial universe. The breadth can fall to 0.1 or less if we create a small-stock index or if the portfolio is highly concentrated.

The effective turnover is proportional to the squared turnover if the portfolio doesn't change constituents at rebalance and to the turnover itself if the portfolio replaces all its constituents. This means that a simple annual turnover can have quite different market impact consequences depending on the structure of the turnover.

Index tilt measures how far the portfolio departs from a volume-weighted portfolio. The volume-weighted portfolio has a tilt of 1; the tilt of any other portfolio is greater than 1.

ENDNOTES

1. The "references" section of a still-valuable review article written 15 years ago mentions about 60 articles. See Donald B. Keim and Ananth Madhavan, "The Cost of Institutional Equity Trades," *Financial Analysts Journal*, vol. 54, no. 4 (July/August 1998), 50-69.
2. Available at <http://www.math.nyu.edu/~almgren/papers/costestim.pdf>. Accessed October 31, 2013.
3. This gap is not always negative; a smart implementation with a reasonable tracking budget may do better than the underlying index.
4. Melanie Zanona, "Index Assets Up 18.4% for Year; Total Worldwide Tops \$7 Trillion," *Pensions & Investments*, September 16, 2013.
5. We prefer to use the post-rebalance portfolio because the weight distribution there is better understood (it is only affected by the strategy, not by the pricing process).
6. Using the universe that corresponds to the cap-weighted benchmark would be the obvious choice, because it is the deviation from those benchmark weights that are of interest.
7. This approximation assumes that prices follow a normal (not log-normal) distribution; clearly, it won't work well if the volatility is sufficiently high. In addition, the stocks that moved down in price by a large amount are more likely to be deleted, further lessening the precision of the estimate.

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