# Stock Market Volatility

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## Cyclicality in Stock Market Volatility and Optimal Portfolio Allocation

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### 10.1 CYCLICALITY IN MARKET VOLATILITY

In standard finance applications, asset class volatilities are usually assumed to be constant over time for simplicity. For example, Markowitz's mean-variance optimization requires that asset class volatilities are known and constant over the holding horizon. While this simplifying assumption reduces the complexity of the models and their calculations, it could also lead to suboptimal portfolio and risk management solutions. If equity market volatility is time varying and is negatively correlated with equity market returns, ignoring

 $<sup>^{\</sup>ast}$   $\dot{\alpha}~$  e authors acknowledge Micah Allred, Vitali Kalesnik, and Lillian Wu for their assistance in completing this chapter.

this countercyclicality could lead to excess allocation to stocks when forward-looking risk for stocks is high. Furthermore, if equity market volatility is positively correlated with the volatilities of other asset classes, ignoring this correlation would again lead to excess allocation to risky assets.

In Table 10.1, we show the U.S equity market volatility in an average bull market versus an average bear market. We use a classic bull/bear market definition, where a bull market is defined as a period of general price appreciation, during which the cumulative market return exceeds 20%. A bear market, by contrast, is a period of price decline, during which the cumulative market negative return exceeds –20%. For simplicity, the market is classified to be in either a bull or bear market phase. Additionally, we show the volatility of other mainstream asset classes over the same equity market cycles. Furthermore, to illustrate the robustness of the finding, we also show, in Table 10.2, the volatilities of these asset classes in different phases of the business cycle (expansion versus recession). We employ the National Bureau of Economic Research (NBER) definitions for expansions and recessions, which uses GDP growth/decline and other macro-economic factors to classify business cycles.

Notice that equity market volatility is significantly higher in bear markets and recessions.  $\dot{\alpha}$  e increase in volatility in down/contracting markets can be attributed to a variety of reasons. Down/contracting markets may be triggered by instability in the macroeconomy. Under this assumption, down/contracting markets are likely to be times where shocks to the productive factors in the economy are more severe and more frequent

Asset Class Volatility (ann.)	Bull	Bear
U.S. equities (S&P 500)	13.33%	17.13%
International equities (MSCI EAFE)	15.54%	16.36%
Bond (Lehman Agg)	5.57%	6.92%
Commodities (DJ AIG)	11.76%	13.83%
Real estate (FTSE NAREIT)	13.01%	15.60%
Asset Class Return (ann.)	Bull	Bear
U.S. equities (S&P 500)	21.09%	-19.09%
International equities (MSCI EAFE)	20.45%	-15.96%
Bond (Lehman Agg)	8.26%	11.29%
Commodities (DJ AIG)	5.14%	-0.69%
Real estate (FTSE NAREIT)	16.57%	2.47%

TABLE 10.1 Asset Class Volatilities over Equity Bull/Bear Market Cycles

Asset Class Volatility (ann.)	Expansion	Recession
U.S. equities (S&P 500)	14.06%	19.00%
International equities (MSCI EAFE)	15.08%	23.24%
Bond (Lehman Agg)	4.84%	10.96%
Commodities (DJ AIG)	11.84%	13.83%
Real estate (FTSE NAREIT)	12.80%	18.84%
Asset Class Return (ann.)	Expansion	Recession
U.S. equities (S&P 500)	14.04%	11.59%
International equities (MSCI EAFE)	15.85%	-2.16%
Bond (Lehman Agg)	7.55%	20.07%
Commodities (DJ AIG)	5.82%	-9.46%
Real estate (FTSE NAREIT)	13.41%	22.25%

TABLE 10.2 Asset Class Volatilities over NBER Expansion/Recession Cycles

Note: A recession is a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales. A recession begins just after the economy reaches a peak of activity and ends as the economy reaches its trough. Between trough and peak, the economy is in an expansion. Expansion is the normal state of the economy; most recessions are brief, and they have been rare in recent decades. à e National Bureau's Business Cycle Dating Committee places particular emphasis on two monthly measures of activity across the entire economy: (1) personal income less transfer payments, in real terms, and (2) employment. In addition, the committee refers to two indicators with coverage primarily of manufacturing and goods: (3) industrial production and (4) the volume of sales of the manufacturing and wholesale-retail sectors adjusted for price changes. à e committee also looks at monthly estimates of real GDP such as those prepared by Macroeconomic Advisers (see http:// www.macroadvisers.com). Although these indicators are the most important measures considered by the NBER in developing its business cycle chronology, there is no fixed rule about which other measures contribute information to the process.

than usual. In down/contracting markets, leveraged investments are likely to face margin calls, which increase liquidity-driven asset sale; these liquidating transactions tend to induce additional price volatility. Lastly, market-making agents and noise traders who engage in market liquidity provision, and who trade against informed flows, are likely to become more risk averse in down/contracting markets. In these markets, where market participants have experienced wealth decline, their ability to bear risks declines as a result (their local risk aversion increases).

From Figure 10.1, we observe that asset class volatilities appear to comove over time, suggesting that common macro factors may drive volatilities for various risky assets. Specifically, we observe from Table 10.1 that the volatilities of other risky asset classes seem to also increase noticeably during equity bear markets.  $\dot{\alpha}$  is increase in volatility suggests that the



FIGURE 10.1 Asset class rolling 36-month volatilities.

increased shocks to equity valuation often spill over to other markets, and that liquidity-driven selling and the reduction in liquidity provision in the capital market are often systemic across various asset classes. Not surprisingly, equity bear markets and recessions can often have significant overlaps and have similar influences on asset return characteristics.

In this chapter, we argue that the countercyclical nature of equity market volatility (high volatility in down markets), combined with positive correlations between asset class volatilities, has a significant impact on optimal portfolio allocation. We first present a simple model of time-varying asset class volatilities. We then illustrate how to calibrate the model and integrate the method with the classic mean-variance approach. We compare our proposed optimal portfolio solution to the standard static portfolio solution where the time-varying volatility is ignored and argue that a dynamic mean-variance approach is superior to the standard approach.

#### 10.2 LITERATURE REVIEW ON MARKET VOLATILITY

Before we introduce our model on cyclical equity market volatility, we explore the literature on market volatility and examine the drivers for the level and variation for market variance. Using a simple present value model, Shiller (1981) finds that the level of stock market volatility is too high relative to the variation in the underlying micro and macro fundamentals.

Specifically, he finds that the changes in real dividends and real interest rates cannot explain the level of market volatility. Studies that examine the variation in market volatility also conclude that standard macro factors and corporate characteristics cannot explain the time-varying nature of equity volatility. Specifically, Officer (1973), Black (1976), and Christie (1982) find that financial leverage only weakly explains the variation in market volatility. Schwert (1989) finds that standard macroeconomic variables, such as inflation, money growth, and industrial production, also do not sufficiently explain the variation in the market volatility. Å erefore, nonfundamentally based volatility drivers likely exist and may have better explanatory powers.

Behavioral finance literature points to information herding (cascading), noise trading, and liquidity-driven transactions as potential reasons for the higher level of market volatility, relative to the volatility in the underlying information flow.  $\dot{\alpha}$  eoretical work by Banerjee (1992) and Bikhchandani et al. (1992) suggests that information cascade can lead to price overshooting, which would inject additional volatility, in excess of the contribution from the existing volatility drivers. Campbell and Kyle (1993) and DeLong et al. (1990) study the effect of noninformed trading (uninformed speculation by noise trader or portfolio trading driven by liquidity shocks to the investor).  $\dot{\alpha}$  ey suggest that these uninformed trading activities create a new source of shocks to prices.  $\dot{\alpha}$  is additionally creates excess equity market volatility.

å e return predictability literature and the value premium literature offer rational pricing models as well as behavioral explanations for timevarying market volatility. Ferson and Harvey (1991) find that expected stock market return and volatility vary over time in a predictable way. Lettau and Ludvigson (2001), Chordia and Shivakumar (2002), and Zhang (2005) offer models that relate variation in aggregate risk aversion to decline in aggregate wealth. Intuitively, a period of negative returns driven by shocks to fundamentals will lead to aggregate wealth destruction; this can increase the aggregate risk aversion, which further decreases prices today and increases forward-looking return and increases volatility contemporaneously.

Equilibrium models of cyclical volatility are often difficult to apply; in addition, they often do not match well to data or offer insufficient degrees of freedom for empirical calibration. For this reason, statistical models are often relied upon for modeling stochastic volatility; these statistical models can be used with great flexibility for asset pricing or asset allocation exercises. Various statistical volatility models have been developed specifically to capture and measure time-varying volatilities. Engle (1982) and Bollerslev (1986) provide the basic framework for such modeling with the ARCH/GARCH process (autoregressive conditional heteroskedasticity/generalized autoregressive conditional heteroskedasticity).  $\dot{\alpha}$  e technique has been applied widely to the estimation of the time-varying equity market volatility. Recent researches have proposed new techniques that could improve forecasting power through the usage of high-frequency tick-by-tick data. Anderson et al. (2001, 2003, 2005) use 5-minute realized volatility with a vector autoregessive model of log standard deviation, which eliminates much of the serial dependence in the volatilities and appears to outperform the traditional ARCH/GARCH specifications. Ghysels et al. (2006) also use higher-frequency data but propose a regression model using a beta weighting function to estimate and forecast volatility.  $\dot{\alpha}$  eir model appears to be easier to parameterize and provides better forecasts against traditional ARCH/GARCH models. Vasilellis and Meade (1996) show that the implied stock volatility from option prices is an efficient forecast for future volatility. Poon and Granger (2003, 2005) show that option-implied volatility provides the best forecast for future volatility; they used option-implied volatility data from the last 20 years and compare against volatility models such as time-weighted volatility, rolling volatility, ARCH/GARCH, and other stochastic volatility models.

So why should we care about time-varying market volatility? If we do not properly characterize the time-varying nature of volatility and covariance for the various capital markets we invest in, our asset pricing model would be flawed, our portfolio allocation would be suboptimal, and our ex ante risk assessment would be incorrect. Bentz (2003) and Bollerslev et al. (1988) show that using a time-varying covariance estimate (beta estimate) can improve the application of the capital asset pricing model for forecasting returns. Horasanh and Fidan (2007) show that applying GARCH estimates for volatility can improve portfolio allocation efficiency. Blake and Timmermann (2002) find evidence that some pension funds seem to vary asset allocation to take advantage of time-varying asset class volatilities and risk premia. Myers (1991) finds that using GARCH models can improve the effectiveness of hedging fixed-income exposure relative to traditional regression approach with constant variance. Baillie and Myers (1991) extend the study into the commodities market and find that GARCH-based hedging provides a substantial improvement in risk reduction effectiveness.

#### 10.3 A SIMPLE MODEL OF TIME-VARYING VOLATILITY

We introduce in this section a simple model that captures the countercyclicality nature of asset class volatilities. à is approach is more intuitive and more tractable than other models of time-varying volatilities and leads to greater intuition and ease of calibration. à e world is assumed to follow a two-state, two-stage Markov chain. à e world can either be in a bull market state (U for upmarkets) or in a bear market state (D for downmarkets) at time *t*. For example, if we are currently in a bull market, for the next period, the economy can either transition into a bear market with the transition probability  $P_{U\rightarrow D}$  or remain in the current bull state with probability  $P_{U\rightarrow U} = 1 - P_{U\rightarrow D}$ . If we transition to the bear market state at time t + 1, then for t + 2, we could transition to the bull market state with probability  $P_{D\rightarrow U}$  or remain in the bear market state with probability  $P_{D\rightarrow U}$ . Figure 10.2 illustrates graphically this Markov process.

Following the empirical results shown in Tables 10.1 and 10.2, the bull market state (U) is characterized by lower volatilities and higher returns for the asset classes, while the bear market state (D) is characterized by high volatilities and lower returns. We let  $\Sigma_U$  denote the vector of bull market volatilities { $\sigma_1^U, \sigma_2^U, ..., \sigma_k^U$ } and  $\Sigma_D$  denote the vector of bear market volatilities { $\sigma_1^D, \sigma_2^D, ..., \sigma_k^D$ }; note that we assume an investment opportunity set with *k* assets. Similarly,  $\mu_U$  and  $\mu_D$  denote the vector of bull and bear market mean returns { $\mu_1^U, \mu_2^U, ..., \mu_k^U$ } and { $\mu_1^D, \mu_2^D, ..., \mu_k^D$ }.



 $\mathbf{P}_{Bull -> Bull} \quad \text{ is the probability of starting in a bull market state and remaining in the bull market state next period.}$ 

 $\mathbf{P}_{Bull->Bear} \quad \text{is the probability of starting in a bull market state and transitioning} \\ \text{to the bear market state next period.}$ 

FIGURE 10.2 A Markov two-state (bull/bear market) transition model.



FIGURE 10.3 Identifying bear market periods (January 1979–December 2007).

#### 10.3.1 Model Parameter Calibration

We now illustrate how to calibrate this Markov model to data. First, we classify our time period into equity bull and bear market periods (using the common definitions of bull and bear markets presented earlier). For the data time span T, we decompose T into nonoverlapping bull/bear time segments as illustrated in Figure 10.3. We denote the bull market time segments as  $\{T_1^U, T_2^U, ..., T_m^U\}$  and the bear market time segments as  $\{T_1^D, T_2^D, ..., T_n^D\}$ , where  $T = \sum_{i=1}^m T_i^U + \sum_{i=1}^n T_i^D$ .  $\dot{\alpha}$  e average duration for an equity bull market is empirically estimated by  $\tau_U = \frac{1}{m} \sum_{i=1}^m T_i^U$ , and the average duration for a bear market is  $\tau_D = \frac{1}{n} \sum_{i=1}^n T_i^D$ . Using S&P 500 return data from January 1976 through June 2008, we have encountered four bear market cycles, each averaging about 17 months, whereas the four bull market cycles average about 81 months each.<sup>\*</sup>

To compute the Markov transition probabilities  $P_{U\to D}$  and  $P_{D\to U}$ , we make use of the derived relationships, where  $P_{U\to U} = 1 - \frac{1}{\tau_U}$  with

<sup>&</sup>lt;sup>\*</sup> Certainly, the more data that are used in the estimation, the more reliable and robust the estimation. Because there have not been many bull/bear market cycles, the estimation error will always be a concern when applying this calibration exercise.

 $P_{U\to D} = 1 - P_{U\to U}$  and  $P_{D\to D} = 1 - \frac{1}{\tau_D}$  with  $P_{D\to U} = 1 - P_{D\to D}$  (see Meyn and Tweedie (1993) for a complete theoretical treatment on Markov models). Again, using data from 1976 through June 2007, conditioning on starting in a bull market, the probability for transitioning to a bear market by next year is  $P_{U\to D} = 15\%$ , and the probability for remaining in a bull market next year is  $P_{U\to U} = 85\%$ . Similarly, conditioning on starting in a bear market next year is  $P_{D\to D} = 27\%$ , and the probability for transitioning to a bull market next year is  $P_{D\to D} = 73\%$ .

For each asset class, the time series of returns  $r = \{r_1, r_2, ..., r_T\}$  is divided into bull market returns  $r_U = \{r_1^U, r_2^U, ...\}$  and bear market returns  $r_D = \{r_1^D, r_2^D, ...\}$ . à e return volatility and expected return corresponding to the bull and bear market cycles are then estimated by the subsample volatility and average return. Using S&P 500 data from 1976 through 2007, the bull market volatility is 13% while the bear market volatility is 17%. à e bull market average return is 21% versus –19% for the bear market average.

#### 10.4 OPTIMAL PORTFOLIO ALLOCATION

With the economy characterized and calibrated as a two-state Markov chain, we are now ready to examine the optimal portfolio exercise. Like the classic Markowitz portfolio analysis, we are seeking a set of portfolio weights that maximize the portfolio expected return given a volatility constraint.  $\dot{\alpha}$  e portfolio optimization requires that we supply the expected returns for all of the assets in the investment opportunity set and the covariance matrix governing returns. In the context of our two-state Markov model, first, we must determine the current state of the economy before we can compute these asset return moments.  $\dot{\alpha}$  is can be a difficult exercise, as we need to identify whether we are currently in a bull or bear market state; there may be no clear evidence suggesting a bull or bear market condition. In the next section, we discuss how to refine the model to overcome this uncertainty in our knowledge regarding the current state of the economy. We continue with the basic model for the time being.

Next, we need to use the calibrated model parameters from the previous section to compute the moments required for mean-variance optimization. Again, recall that we have *k* assets. In our simple model, we have two possible future states with conditional probability  $P_{S->U}$  of transitioning to a bull market from the current state *S* and  $P_{S->D}$  of transitioning to a bear market  $\dot{a}$  e expected return vector and covariance matrix depends upon the future regime. Let  $\mu_U$  and  $\mu_D$  each be a  $1 \times k$  vector of expected returns, and  $\Omega_U$  and  $\Omega_D$  be the covariance matrix for the bull and bear states, respectively.  $\dot{a}$  e vector of expected asset returns given that we are in state *S* is  $\mu(S) = P_{S->D} \mu_U + P_{S->D} \mu_D$ .

ά e derivation of the covariance term is a bit more complex. We are interested in computing  $\Omega(S) = E[(r-\mu)'(r-\mu)|S]$ . From the law of iterated expectations:

$$E[(r-\mu)'(r-\mu)|S]$$
  
=  $P_{S->U}E[(r-\mu)'(r-\mu)|U] + P_{S->D}E[(r-\mu)'(r-\mu)|D]$   
=  $P_{S->U}E[(r_u-\mu)'(r_u-\mu)|U] + P_{S->D}E[(r_D-\mu)'(r_D-\mu)|D]$ 

To simplify the above expression, we note

$$\begin{split} E[(r_U - \mu)'(r_U - \mu) | S] \\ &= E[(r_U - \mu_U + P_{S - >D}(\mu_U - \mu_D))'(r_U - \mu_U + P_{S - >D}(\mu_U - \mu_D))] \\ &= \Omega_U + P_{S - >D}^2(\mu_U - \mu_D)'(\mu_U - \mu_D) \end{split}$$

 $\dot{\alpha}$  e covariance matrix then becomes:

$$\Omega(S) = P_{S \to U} \Big[ \Omega_U + P_{S \to D}^2 (\mu_U - \mu_D)' (\mu_U - \mu_D) \Big]$$
$$+ P_{S \to D} \Big[ \Omega_D + P_{S \to U}^2 (\mu_D - \mu_U)' (\mu_D - \mu_U) \Big]$$

#### 10.5 SIMPLE MODEL EXTENSION

We noted previously that it may be difficult to determine exactly the current state of the economy. Generally, one does not know with a high degree of certainty whether one is in a bull market or bear market state (until after the market has fully run its course, which would eliminate the information advantage of this approach).

ά e lack of perfect knowledge about the current state means that we need to adjust for this uncertainty in our calculation. Hsu and Kalesnik (2008) show the benefits of properly adjusting for model uncertainty in portfolio construction and risk management. Suppose that there is a probability  $P_U$  that we are in a bull market environment, and  $P_D = 1 - P_U$  that we are in a bear market environment.  $\dot{\alpha}$  ese probabilities will likely depend on a set of macroeconomic observables; as the macro variables change over time, the probabilities will also shift.  $\dot{\alpha}$  e computation of the asset class return moments becomes more involved now; first, we need to repeat the exercise described in the last section for the bull and bear market states independently.  $\dot{\alpha}$  en we formulate a model for characterizing  $P_U$  and  $P_D$ .  $\dot{\alpha}$  e uncertainty-adjusted moments for the mean-variance optimization are then computed as  $\mu = P_U \mu(U) + P_D \mu(D)$  and  $\Omega = P_U \Omega(U) + P_D \Omega(D)$ . Finally, the mean-variance optimal portfolio is determined by the standard Markowitz optimal portfolio solution.

Since the probabilities  $P_U$  and  $P_D$  change in response to the changes in the macroeconomy, the optimal portfolio also changes with observed changes in the macro variables. As we observe signs that suggest greater likelihood that we have entered a bear market,  $P_D$  will increase and the optimal portfolio will take on a lower risk posture given the potentially higher volatility and lower forward returns.

#### 10.6 CONCLUSION

Equity market volatility is time varying, as is the equity risk premium. Additionally, other risky asset volatilities appear to also be time varying and positively correlated with equity market volatility. Specifically, we find that volatilities for various risky asset classes tend to be low in equity bull markets and high in equity bear markets. Capturing this time-varying characteristic of joint asset class volatilities is important in order to properly execute mean-variance portfolio optimization.

We introduce in this chapter a simple and intuitive model of timevarying volatility and risk premia using the Markov state switching modeling technique. In our simple model, the state of economy switches between bull and bear markets. Asset classes have distinct volatility and risk premium characteristics in the two states of the market. By properly formulating the conditional moments, the traditional mean-variance optimization becomes a conditional optimization, and the traditional static optimal portfolio solution becomes a dynamic one.  $\dot{\alpha}$  is results in a more efficient asset allocation, which takes advantage of the time-varying nature of market risk characteristics.

Applying this simple modeling technique improves portfolio characteristics over time. In the traditional constant volatility and risk premium model, optimal portfolio allocation remains constant over time. Å e state switching modeling approach has significant advantages when market volatilities and risk premia are time varying. Specifically, when we are in a state of bull equity market, where volatility has been low, properly assessing the probability for transitioning into a bear equity market, where the volatility would be substantially higher, would lead to a risk reduction portfolio. Reciprocally, in a bear market state, this approach would suggest greater risk taking. Relative to classic constant volatility models and static portfolio solutions, the time-varying approach with its associated dynamic optimal portfolio solution leads to better long-term portfolio efficiency and therefore a higher portfolio Sharpe ratio.

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