by Roger G. Clarke and Robert D. Arnott

The Cost of Portfolio Insurance: Tradeoffs and Choices

Portfolio insurance can protect a portfolio from declining in value during down markets; in up markets, however, an insured portfolio will not increase in value by as much as a comparable uninsured portfolio. This loss in upside capture represents the most important cost of portfolio insurance. It is sensitive to many factors, some of which can be controlled by the investor. The investor can, for example, reduce the cost of portfolio insurance by (1) lowering the floor return of the insurance strategy, (2) decreasing the percentage of the portfolio's assets covered by insurance, (3) increasing the risk (beta) of the underlying portfolio or (4) extending the insurance strategy's horizon beyond one year.

The investor should also be aware, however, of other factors that are beyond his control. A decline in the risk-free rate, for example, will decrease the implicit return available in the hedged portion of the portfolio, hence increase the cost of insurance. Similarly, an increase in the return on equities relative to the risk-free rate will increase the cost of insurance. Increased volatility of the risky assets will also raise insurance costs by increasing the likelihood of "whipsaw" movements that force the portfolio into selling as the market drops and buying as it rises.

Finally, an insurance strategy, by increasing portfolio turnover, tends to increase transaction costs. Careless trading or market illiquidity translates directly into reduced return, hence increased cost.

ONE OF THE MOST INTERESTING and widely discussed investment management techniques to emerge in recent years is the concept of portfolio insurance. Studies of portfolio insurance usually summarize the impact insurance has on expected return and standard deviation. An examination of the effects on the entire return distribution is also instructive, because many other return attributes are substantively affected by insurance.

This article summarizes the tradeoffs and choices an investor faces in designing a portfolio insurance strategy and reviews a few potential pitfalls. Some of the factors that have a bearing on the performance of portfolio insurance can be directly controlled by the investor; some cannot. It is important for the investor to understand the implications of the choices he makes as well as the implications of those factors he cannot control.

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The Study

Many studies of portfolio insurance have used historical results to explore the investment tradeoffs involved.[1] These studies serve a very important role in suggesting the historical behavior of protection strategies. However, they may not offer an appropriate guide to the *costs* of portfolio insurance, because their results are highly dependent on the specific time period covered by the research. Also, historical research usually provides inadequate precision in defining the *shape* of the return distribution.[2] For this kind of information, one has to turn to theoretic or stochastic modeling.

To examine the impact of various insurance

1. Footnotes appear at end of article.

choices, we used an algorithm developed by Bookstaber and Clarke to describe the *ex ante* return distribution of a combined portfolio of risky assets and option positions.[3] The examples we use are not meant to detail all possible tradeoffs that can be made in designing an insurance strategy. In practice, however, the cost of insurance will usually depend on the level of interest rates, the realized volatility of the market, the transaction costs encountered and the investor's skill in implementing the dynamic hedge, to name only a few of the factors. These are some of the issues we explore.

Although we describe some of the tradeoffs that exist in structuring an insurance program, it is difficult to recommend which choices are best for a particular individual. The only complete formal framework for choice among risky alternatives is provided by utility theory. Work by Leland illustrates the attractiveness of insurance to investors whose risk tolerance increases with wealth more rapidly than average.[4] Unfortunately, evaluating the specific tradeoffs discussed here would require more detail about the investor's utility function and preferences. This analysis describes some of the tradeoffs that can be made, while leaving a discussion of the optimal choice to another occasion.

The Basic Shape of Portfolio Insurance

The chief characteristic of portfolio insurance is its ability to provide downside protection for the value of a risky asset portfolio while preserving much of the upside potential. The payoff pattern for this strategy offers the same results as holding a portfolio of risky assets and buying a protective put option with an exercise price adequate to achieve the desired floor.[5]

Portfolio insurance can be created in several ways.[6] In practice, most insurers use a dynamic hedging approach, changing the effective exposure of the risky assets in the portfolio by using the futures markets.[7] This dynamic strategy is usually designed to approximate the results that would be obtained by purchasing a put option on the portfolio. The strategy, commonly called a synthetic put option, allows one to create insurance with longer time horizons than the maturities of actual put options would allow.

Not all portfolio insurance strategies, of course, are based on the creation of a synthetic put option.[8] This article, however, focuses explicitly on the factors that affect the return characteristics of a synthetic put strategy. Those factors will have similar effects on most other insurance strategies. The discussion is thus relevant to nearly all types of strategies.

An Example

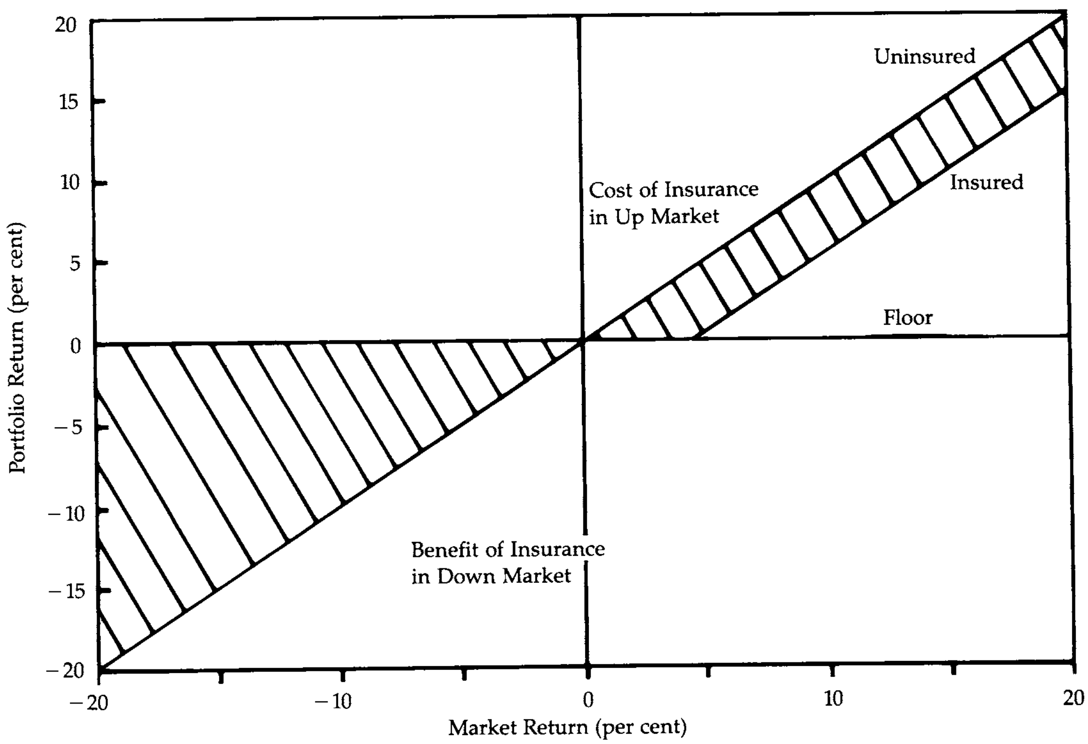
Suppose the investor begins with a portfolio composed of 100 per cent equity invested in an index fund. We will assume that the riskless rate equals 8 per cent and the total expected return on the index fund is 15 per cent, with a volatility of 18 per cent. The fund has a dividend yield of 3 per cent.[9] Insurance is implemented through the purchase of a put option, with a floor equal to the current value of the portfolio; the put option is priced consistent with a Black-Scholes model adjusted for dividends.

Figure A illustrates one way to consider the basic tradeoffs involved in portfolio insurance. The horizontal line in the figure is drawn at the floor, while the diagonal line represents the payoff from the uninsured equity portfolio. The kinked line represents the return profile on the insured portfolio. When the return on the underlying portfolio is below the floor, the insured portfolio will give a return greater than that of the underlying stocks but less than the riskless rate. For higher returns, the insured portfolio will underperform the uninsured portfolio.

The important tradeoff for the investor is the sacrifice of some upside potential for downside protection. The desirable characteristics of insurance are not free; the investor can expect to pay something for the protection. This is represented by the reduced return on the insured portfolio in an up market. This opportunity cost is the loss in upside capture and is approximately equal to the initial cost of the put option. The loss of upside capture is one way of looking at the cost of portfolio insurance.

While Figure A shows the pattern of returns for the insured portfolio, Figure B shows the impact of a portfolio insurance strategy on the probability *distribution* of returns. Line a shows the return distribution for the uninsured portfolio, based on the assumption that the returns for the underlying equity portfolio are log-normally distributed, with a mean return of 15 per cent and a standard deviation of 18 per cent. Line b shows the effect that a simple one-year portfolio insurance strategy has on the probability distribution. As expected, performance below the intended floor return is eliminated. Much of the upside performance of the portfolio is retained,

Figure A Insured vs. Uninsured Portfolio Return



although it is diminished somewhat by the cost of the insurance.

Statistical Attributes

Table I summarizes the statistical attributes of a portfolio insurance strategy. Most of the other tables are based on this format, so it is useful to review the material in some detail.

The *mean return* for an insured portfolio with a one-year horizon and a floor return of 0 per cent decreases by about 2.0 per cent relative to the return on a comparable uninsured portfolio. This decrease in arithmetic return is a second way to describe the cost of portfolio insurance. In essence, an insurance premium is paid in good years as the insured portfolio lags the uninsured portfolio. This loss in upside capture from just the put premium alone amounts to approximately 4.1 per cent, which is partially offset by the protection in bad years, giving a net expected long-term cost of 2.0 per cent per annum.

A third way to describe the cost of portfolio insurance is to look at its impact on the long-run growth or geometric return of the portfolio over repeated insurance programs. The geometric return, g, can be approximated from the arithmetic mean, r, and standard deviation, σ, as follows:

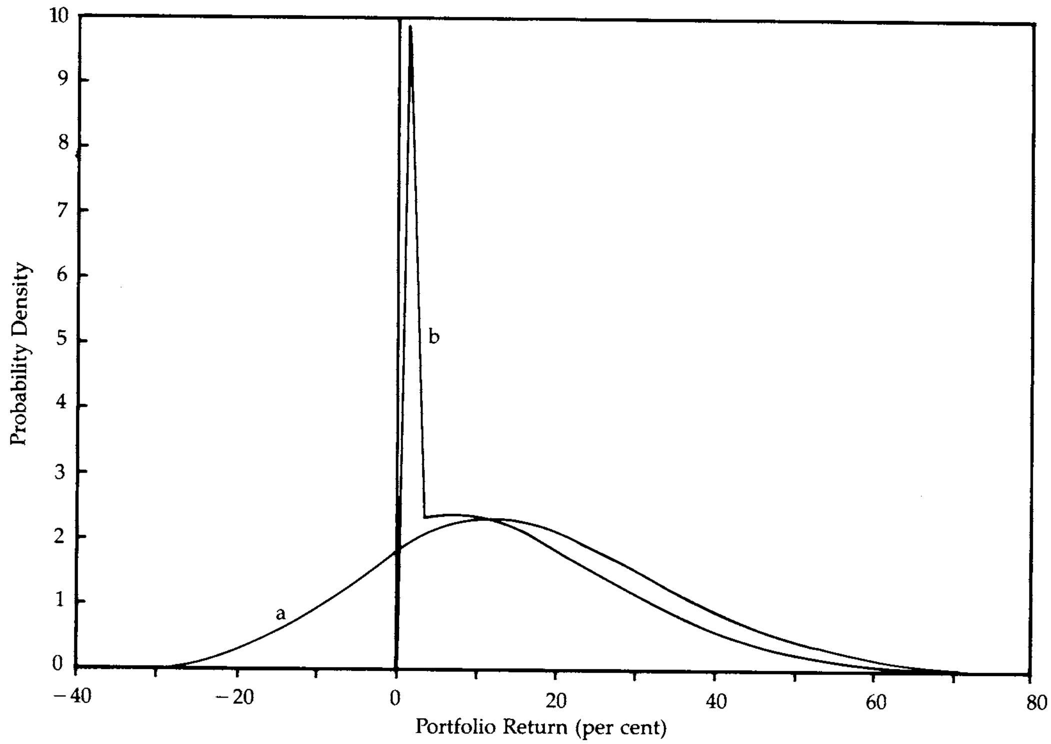
$$g \approx ((1+r)^2 - \sigma^2)^{1/2} - 1.$$

Using this approximation, the geometric return declines from 13.6 per cent to 12.1 per cent with insurance, resulting in a long-run cost of 1.5 per cent per year, compared with the 2.0 per cent cost in the single-period arithmetic mean. The reduction in variance that occurs with the use of portfolio insurance has a slightly positive effect relative to the decrease in arithmetic return, and this lowers the cost of insurance if the program is used repeatedly over the long run.

The loss of upside capture can be illustrated in another way by looking at its effect on *median return*. With insurance, the median return declines much more than the mean return; at least half of the time, the insured portfolio will underperform the uninsured portfolio. In this example, the median return declines from 13.6 to 9.0 per cent.

In essence, the insured portfolio sacrifices 4.1

Figure B Return Distribution



per cent of terminal value in years with a positive return, in exchange for the benefit of a protected return in down markets. This is characterized in the tables as "loss of upside capture."

Naturally, the average beta and the standard deviation decline by a substantial margin with insurance. By shifting exposure from the stock market into a "risk-free" investment in adverse market conditions, insurance decreases the average beta. The beta of an insured portfolio in the short run will vary from 0.00 to 1.00, de-

Table I Return Characteristics for Insured Portfolios with Various Floor Returns

		Insured Floor Return				
	Uninsured	*+5%*	*0%*	*−5%*	*−10%*	*−15%*
Mean Return (%)	15.0	11.1	13.0	14.0	14.5	14.8
Median Return (%)	13.6	5.0	9.0	11.5	12.7	13.3
Geo. Mean Ret. (%)	13.6	10.6	12.1	12.9	13.2	13.4
Avg. Beta	1.00	0.49	0.75	0.88	0.95	0.98
St. Dev. (%)	18.0	10.2	14.2	16.1	17.1	17.6
Skewness	0.47	2.1	1.24	0.88	0.69	0.57
Loss of Upside Capture (%)	0.0	10.2	4.1	1.8	0.8	0.3
Arith. Mean Cost (%)	0.0	3.9	2.0	1.0	0.5	0.2
Geo. Mean Cost (%)	0.0	3.0	1.5	0.7	0.4	0.2
Probability of Return Less Than:						
0%	20.7	0.0	0.0	24.2	22.2	21.2
Riskless Rate	37.3	61.9	47.6	41.9	39.2	38.0

Assumptions

Dividend Yield	= 3.0%	Riskless Rate	= 8.0%
Expected Market Return	= 15.0%	Percentage of Portfolio Insured	= 100%
Market Volatility	= 18.0%	Time Horizon	= 1 Year

Table II Probability Distributions of Return for Insured Portfolios with Various Floor Returns

Return Range	Uninsured Portfolio	Insured Floor Return +5%	0%	−5%	−10%	−15%
Below −40%	0	0	0	0	0	0
−40−−30	0.1	0	0	0	0	0
−30−−20	1.1	0	0	0	0	0
−20−−10	5.5	0	0	0	0	7.0
−10–0	13.9	0	0	24.2	22.1	14.2
0–10	21.3	66.3	52.3	22.3	21.7	21.4
10–20	21.9	17.4	20.8	21.6	21.8	21.9
20–30	16.9	9.6	14.0	15.6	16.4	16.7
30–40	10.3	4.3	7.5	9.1	9.8	10.1
40–50	5.3	1.6	3.4	4.3	4.9	5.1
50+	3.7	0.8	2.0	2.9	3.3	3.6
	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Assumptions

Dividend Yield	= 3.0%	Riskless Rate	= 8.0%
Expected Market Return	= 15.0%	Percentage of Portfolio Insured	= 100%
Market Volatility	= 18.0%	Time Horizon	= 1 Year

pending on market conditions; it will average 0.75 over the full range of market returns. The standard deviation is similarly reduced, from 18.0 to 14.2 per cent.

By eliminating the negative end of the return spectrum, insurance increases the "skewness" of the portfolio. In essence, this means that the distribution of returns is not symmetric; there is a greater likelihood of very strong results than of very weak results. The natural skewness of the log-normal return distribution increases from 0.47 to 1.24.

Finally, the probability of a return less than the 0 per cent floor return is cut from 20.7 per cent to zero with insurance. *This is the entire purpose of portfolio insurance, to eliminate the risk of losses below the designated floor.* The investor pays for this by accepting an increased likelihood of only modest returns. The likelihood of returns below the risk-free rate rises from 37 per cent to nearly 48 per cent with insurance, and the median return decreases. In essence, the investor cannot get "something for nothing." The shift in probabilities is underscored in Table II: Probability is shifted from returns both above and below the floor to the range just above the floor.

Controlling Insurance Risks and Rewards

The investor can alter the basic insurance pattern in several ways in order to control directly or indirectly the cost and return characteristics of an insured portfolio.

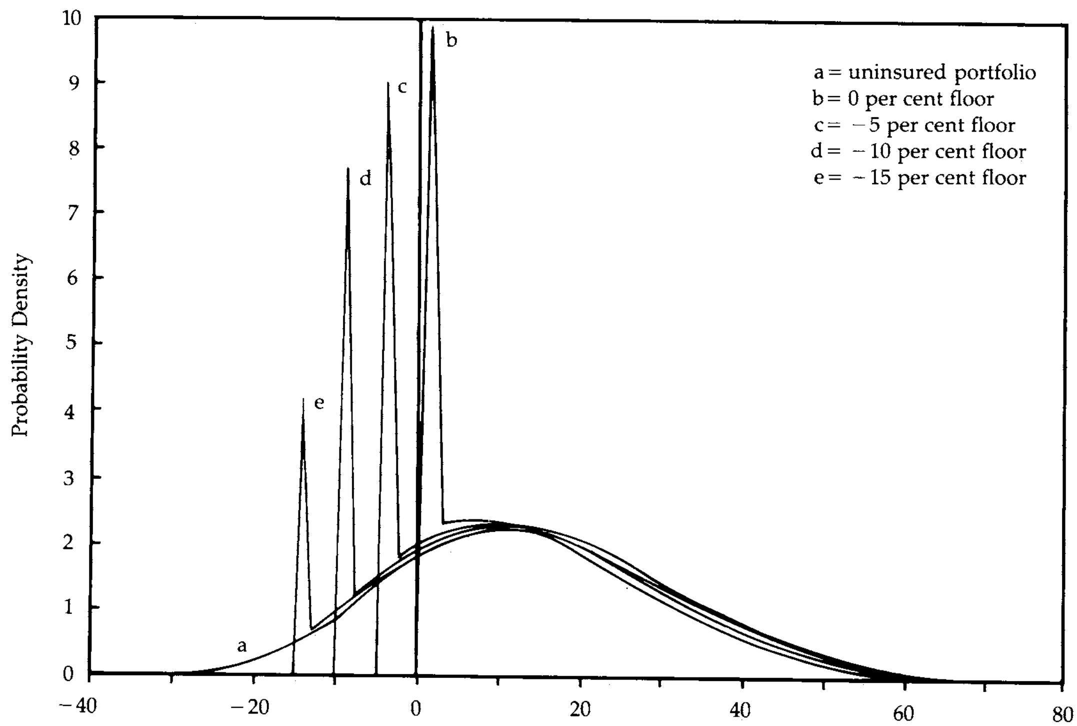
Adjusting Floor Return

If a homeowner wants to reduce the cost of an insurance policy, he can easily do so by raising the deductible on the insurance. The analogy for portfolio insurance is to reduce the target floor return. If the investor seeks to assure that the return will not fall below −10 per cent, the expected cost of the insurance will be much less than the cost with a 0 per cent floor.

Figure C illustrates the tradeoffs available by adjusting the floor return. As the floor return declines, less and less of the portfolio is protected, but more upside potential is captured, as the sharp increase in the median indicates. As a result, the insurance costs less. With a very low insurance floor, such as −15 per cent, the impact of protection on the return characteristics of the portfolio is very modest, but the worst-case catastrophic loss is still averted. Line a represents the uninsured portfolio; lines b, c, d and e represent floor returns of 0, −5, −10 and −15 per cent, respectively.

Table I summarizes these results statistically. All three measures of the cost of portfolio insurance decline as the insurance floor decreases from +5 to −15 per cent. The loss of upside capture decreases from 10.2 to 0.3 per cent. The arithmetic mean cost decreases from 3.9 to 0.2 per cent, and the long-run geometric mean cost decreases from 3.0 to 0.2 per cent. As the likelihood of returns below −15 per cent on an uninsured portfolio is comparatively modest, the cost of insurance is negligible if the floor return is set at or below this very low threshold.

Figure C Return Distributions for Various Floor Returns



It should also be noted that the impact of further decreases in the floor return is minimal.[10]

Coinsurance

An alternative way to reduce the cost of insurance is by insuring only a portion of the assets. In the insurance industry, this is sometimes called "coinsurance." With this strategy, the investor chooses to bear some of the risk of loss by applying an insurance strategy to only a portion of the entire portfolio. Naturally, the investor bears fully any loss on the uninsured portion.

Figure D illustrates the effect of coinsurance. As the investor bears more of the risk of the portfolio, he receives less protection but retains more upside potential. This reduces the expected cost of a portfolio insurance strategy, but increases the likelihood of significant losses. Even so, this strategy still reduces the likelihood of catastrophic losses. Line a represents the uninsured portfolio; lines b, c and d represent, respectively, 25, 50 and 75 per cent insurance.

In a year like 1974, with the market down 26 per cent, coinsurance in which protection with a 0 per cent floor is applied to half the portfolio would lose a far more tolerable 13 per cent. The cost of coinsurance varies with the proportion of the portfolio insured. The arithmetic mean cost increases from 0.5 to 2.0 per cent as the insured proportion of the portfolio increases from 25 to 100 per cent. Similar increases in cost are reflected in the loss of upside capture and the geometric mean cost.

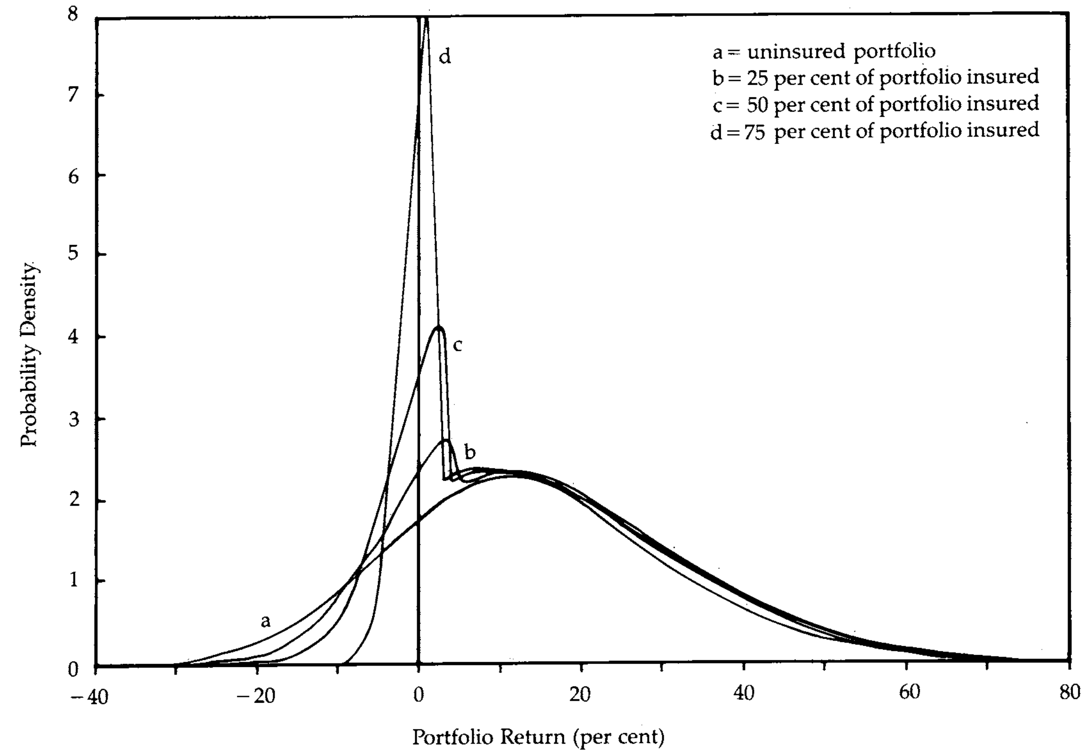
Table III describes the effects of coinsurance statistically for various levels of coverage. The degree of coinsurance correlates directly with the rewards of a portfolio insurance process.

Changing Portfolio Risk

Many practitioners of insurance strategies have argued that protection is ideally suited for use as a safety net in conjunction with aggressive exposure to the high-return (and high-risk) asset classes.[11] This rationale makes good sense *if* the increase in the rate of return is acceptable, given the change in risk.

Figure E illustrates some of the tradeoffs involved with this strategy. The increase in risk for the underlying assets is reflected by an

Figure D Return Distributions for Various Percentages of Portfolio Insured



increase in the underlying portfolio's beta. Line a represents the uninsured portfolio with a beta of 1.0. Lines b, c and d represent insured portfolios with betas of 1.0, 1.5 and 2.0, respectively. Table IV gives the summary statistics for various changes in beta. Note particularly the contrast between the uninsured equity portfolio and an insured portfolio with an underlying beta of 1.75. Both portfolios have an average beta of approximately 1.0, but the risk profiles are quite different. Note also the portfolio with a beta of 1.50; both the uninsured equity fund and

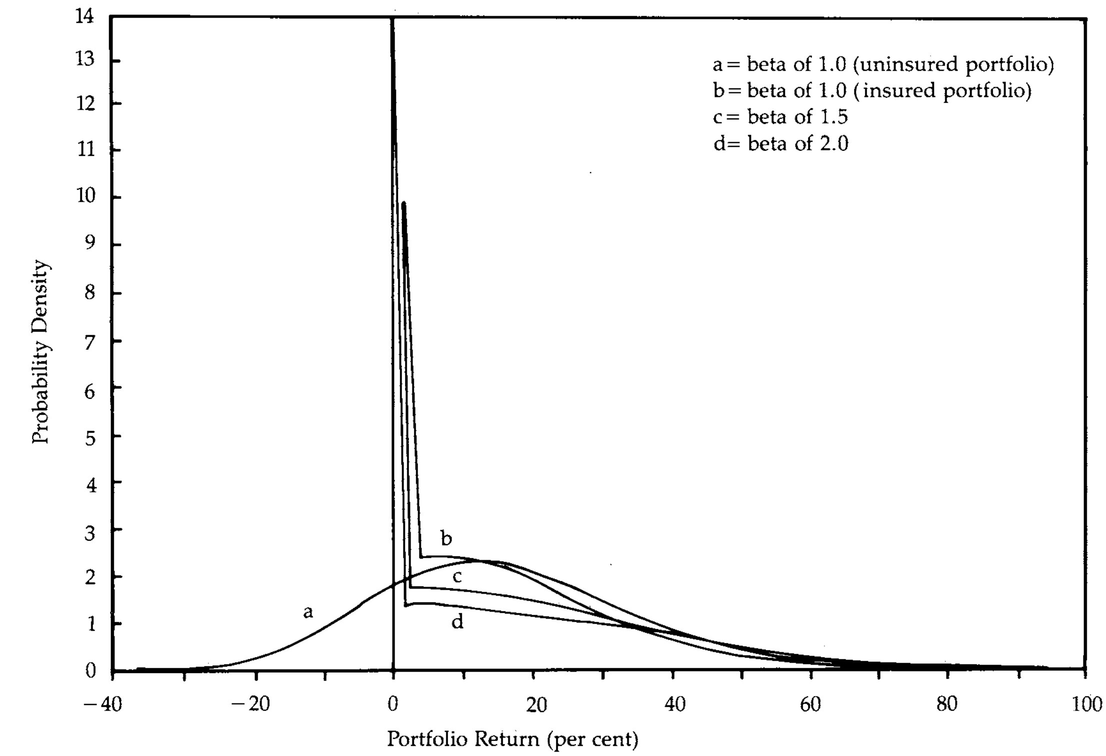
Table III Return Characteristics for Insured Portfolios with Various Fractions of Portfolio Covered by Insurance

	Percentage of Portfolio Insured				
	0%	*25%*	*50%*	*75%*	*100%*
Mean Return (%)	15.0	14.5	14.0	13.5	13.0
Median Return (%)	13.6	12.5	11.4	10.2	9.0
Geo. Mean Ret. (%)	13.6	13.2	12.9	12.5	12.1
Avg. Beta	1.00	0.94	0.88	0.82	0.75
St. Dev. (%)	18.0	16.9	15.9	15.0	14.2
Skewness	0.47	0.68	0.89	1.08	1.24
Loss of Upside Capture (%)	0.0	1.0	2.1	3.1	4.1
Arith. Mean Cost (%)	0.0	0.5	1.0	1.5	2.0
Geo. Mean Cost (%)	0.0	0.4	0.7	1.1	1.5
Probability of Return Less Than:					
0%	20.7	20.7	20.7	20.7	0.0
Riskless Rate	37.3	39.7	42.3	44.9	47.6

Assumptions

Dividend Yield	= 3.0%	Riskless Rate	= 8.0%
Expected Market Return	= 15.0%	Insured Floor Return	= 0%
Market Volatility	= 18.0%	Time Horizon	= 1 Year

Figure E Return Distributions for Various Betas of Underlying Portfolio



the insured, levered portfolio have standard deviations of 18.0 per cent but, again, the probability distributions are quite different.

The arithmetic mean return of the insured portfolio rises with the increase in leverage. But the investor contemplating this strategy should be fully cognizant of the changes in other attributes of the return distribution. The most note-

Table IV Return Characteristics for Insured Portfolios with Various Leverage (Beta) on Underlying Holdings

	Uninsured	*Insured*					
		Beta of Underlying Portfolio					
	1.00	*0.75*	*1.00*	*1.25*	*1.50*	*1.75*	*2.00*
Mean Return (%)	15.0	12.2	13.0	13.6	14.0	14.3	14.4
Median Return (%)	13.6	9.9	9.0	7.8	6.1	4.2	2.1
Geo. Mean Ret. (%)	13.6	11.6	12.1	12.4	12.5	12.6	12.4
Avg. Beta	1.00	0.63	0.75	0.86	0.94	1.01	1.06
St. Dev. (%)	18.0	11.6	14.2	16.3	18.2	19.8	21.1
Skewness	0.47	1.03	1.24	1.42	1.56	1.69	1.81
Loss of Upside Capture (%)	0.0	2.1	4.1	6.7	9.7	13.1	16.7
Arith. Mean Cost (%)	0.0	2.8	2.0	1.4	1.0	0.7	0.6
Geo. Mean Cost (%)	0.0	2.0	1.5	1.2	1.1	1.0	1.2
Probability of Returns Less Than:							
0%	20.7	0.0	0.0	0.0	0.0	0.0	0.0
Riskless Rate	37.3	44.1	47.6	50.5	53.2	55.5	57.6

Assumptions

Dividend Yield	= 3.0%	Percentage of Portfolio Insured	= 100%
Expected Market Return	= 15.0%	Insured Floor Return	= 0%
Market Volatility	= 18.0%	Time Horizon	= 1 Year
Riskless Rate	= 8.0%		

Table V Return Characteristics for Insured Portfolios with Various Investment Horizons

	Portfolio Insurance Horizon					
	One Year		*Two Years*		*Three Years*	
	Uninsured	*Insured*	*Uninsured*	*Insured*	*Uninsured*	*Insured*
Mean Total Return (%)	15.0	13.0	32.2	30.6	52.1	51.3
Median Total Return (%)	13.6	9.0	29.9	27.4	49.0	48.1
Geo. Mean Ret. (%)	13.6	12.1	29.7	28.4	48.9	48.2
Avg. Beta	1.00	0.75	1.0	0.92	1.00	0.98
St. Dev. (%)	18.0	14.2	25.5	23.7	31.1	30.6
Skewness	0.47	1.24	0.58	0.85	0.62	0.69
Loss of Upside Capture (%)	0.0	4.1	0.0	1.9	0.0	0.6
Arith. Mean Total Cost (%)	0.0	2.0	0.0	1.6	0.0	0.8
Geo. Mean Total Cost (%)	0.0	1.5	0.0	1.3	0.0	0.7
Probability of Return Less Than:						
0%	20.7	0.0	8.6	0.0	2.5	0.0
Riskless Rate	37.3	47.6	28.7	32.2	20.4	21.3

Assumptions

Dividend Yield	= 3.0%	Percentage of Portfolio Insured	= 100%
Expected Market Return	= 15.0%	Insured Floor Return	= 0%
Market Volatility	= 18.0%	Time Horizon	= 1 Year
Riskless Rate	= 8.0%		

worthy is the sharp decrease in the median return, reflecting the increased probability of returns below the riskless rate. For example, the median return for the insured portfolio with an underlying beta of 2.0 declines all the way to 2.1 per cent, reflecting the fact that there is a 57.6 per cent probability that the portfolio's return will be less than the riskless rate of 8 per cent.

It is also interesting to note the impact on the geometric mean return from increases in leverage. At first, the geometric mean return increases with the increase in beta, but it reverses itself above a beta of 1.75 and actually decreases. The result is a decrease in the geometric mean cost of the insurance at lower risk levels, then a net increase at the higher levels of risk.

These comparisons illustrate an important point that is often overlooked: The traditional method of comparing risk and return by using just mean and standard deviation or mean and average beta is not well suited to return distributions altered by the use of options.[12] Option instruments can change the shape of the probability distribution in drastic ways; in particular, with options the probability distributions are not symmetric anymore. The simple measures of mean and standard deviation do not usually give a complete description of risk-return tradeoffs when distributions are asymmetric.

Extending the Protection Horizon

An indirect way to change the level of protection is to change the protection horizon. With a one-year floor return of 0 per cent, it would require a return only 15 per cent below expectations to "hit the floor." If we extended this same level of protection over two or three years, it would require total shortfalls of some 30 or 45 per cent, respectively, to "hit the floor." Extending the portfolio insurance horizon is an indirect way to reduce the degree of protection, and thereby reduce the cost of insurance.

The cost reduction associated with extending the protection horizon is fairly dramatic. To assure a three-year floor of at least 0 per cent would require three separate 0 per cent portfolio insurance programs. The expected cost would be 200 basis points annually. If the investor has a longer investment horizon than one year, this cost can be brought down to 80 basis points over the three-year horizon, or an average of 27 basis points per year. Yet the investor still achieves the same objective. Table V illustrates the effects of extending the protection horizon.

If an investor has unusual circumstances that require a short protection horizon, this discussion may be moot. For example, if a pension executive has a board that expects positive returns each year, regardless of market movements, than a one-year insurance strategy may be the only prudent course. If the horizon of the underlying portfolio is a long-term one, however, one-year insurance may help the investor to achieve a very costly objective (namely, positive returns each year) that is irrelevant for the long-term needs of the portfolio.

Table VI Effect of Riskless Rate on Return Characteristics of Insured Portfolios

	Riskless Rate							
	6%		*8%*		*10%*		*12%*	
	Uninsured	*Insured*	*Uninsured*	*Insured*	*Uninsured*	*Insured*	*Uninsured*	*Insured*
Mean Return (%)	13.0	10.3	15.0	13.0	17.0	15.5	19.0	17.9
Median Return (%)	11.6	5.2	13.6	9.0	15.6	12.3	17.7	15.3
Geo. Mean Ret. (%)	11.6	9.5	13.6	12.1	15.6	14.5	17.6	16.8
Avg. Beta	1.00	0.67	1.00	0.75	1.00	0.81	1.00	0.86
St. Dev. (%)	18.0	12.9	18.0	14.2	18.0	15.1	18.0	15.8
Skewness	0.48	1.49	0.47	1.24	0.47	1.06	0.46	0.92
Loss of Upside Capture (%)	0.0	6.0	0.0	4.1	0.0	2.9	0.0	2.0
Expected Insurance Cost (%)	0.0	2.7	0.0	2.0	0.0	1.5	0.0	1.1
Geo. Mean Cost (%)	0.0	2.1	0.0	1.5	0.0	1.1	0.0	0.8
Probability of Return Less Than:								
0%	24.5	0.0	20.7	0.0	17.1	0.0	14.0	0.0
Riskless Rate	37.3	51.9	37.3	47.6	37.2	44.5	37.2	42.3

Assumptions

Dividend Yield	= 3.0%	Percentage of Portfolio Insured	= 100%
Expected Market Return	= Riskless Rate + 7% (Equity Risk Premium)	Insured Floor Return	= 0%
Market Volatility	= 18.0%	Time Horizon	= 1 Year

Changing Market Conditions

We have reviewed four factors the investor can control directly. Some factors are out of the investor's control, but their effects should not be ignored. Conditions in the capital markets, for example, can affect the expected cost of portfolio insurance significantly. A decline in the risk-free rate or an increase in the equity risk premium or in market volatility are, in particular, likely to increase the cost of portfolio insurance.

Table VI details the impact of changes in the riskless rate on the cost of portfolio insurance. An increase in the riskless rate sharply reduces the likelihood of achieving only the floor return. This results in part because the uninsured portfolio is expected to have a higher return; hence it takes a greater shortfall, vis-a-vis expectations, for the uninsured portfolio to fall below the target floor. The probability of negative returns drops from 24.5 to 14.0 per cent as the

Table VII Effect of Equity Risk Premium on Return Characteristics of Insured Portfolios

	Equity Risk Premium					
	5%		*7%*		*9%*	
	Uninsured	*Insured*	*Uninsured*	*Insured*	*Uninsured*	*Insured*
Mean Return (%)	13.0	11.5	15.0	13.0	17.0	14.5
Median Return (%)	11.6	6.9	13.6	9.0	15.6	11.2
Geo. Mean Ret. (%)	11.6	10.7	13.6	12.1	15.6	13.6
Avg. Beta	1.00	0.71	1.00	0.75	1.00	0.79
St. Dev. (%)	18.0	13.6	18.0	14.2	18.0	14.7
Skewness	0.48	1.38	0.47	1.24	0.47	1.12
Loss of Upside Capture (%)	0.0	4.3	0.0	4.1	0.0	4.0
Arith. Mean Cost (%)	0.0	1.5	0.0	2.0	0.0	2.5
Geo. Mean Cost (%)	0.0	0.9	0.0	1.5	0.0	2.0
Probability of Return Less Than:						
0%	24.5	0.0	20.7	0.0	17.1	0.0
Riskless Rate	41.8	52.5	37.3	47.6	32.8	42.6

Assumptions

Dividend Yield	= 3.0%	Market Volatility	= 18.0%
Riskless Rate	= 8.0%	Percentage of Portfolio Insured	= 100%
Expected Market Return	= 8% + Equity Risk Premium	Insured Floor Return	= 0%
		Time Horizon	= 1 Year

riskless rate rises from 6 to 12 per cent. This means that a put option is further "out of the money" at high riskless rates than at low rates; "out of the money" options cost less than "at the money" options.

A lower floor return, as noted, also reduces the cost of portfolio insurance by a substantial margin. An increase in the riskless rate effectively lowers the insured portfolio's floor vis-a-vis the uninsured expected return. The increased return available for the risky asset means greater room for adverse performance before a significant hedge position is required; the resulting increase in the potential for upside capture reduces the cost of insurance. *This relation between the riskless rate and the cost of portfolio insurance is often overlooked.*

A change in the equity risk premium also affects the cost of insurance, as Table VII shows. The equity risk premium of 7 per cent used in all our previous exhibits is commonly used in the investment industry and is in line with long-term historical patterns.[13] However, the expected equity risk premium will change from time to time as investors' tolerance for risk changes. The 7 per cent figure can only be viewed as a ballpark estimate of the future risk premium.

As Table VII indicates, the cost of portfolio insurance increases with an increase in the equity risk premium. If investors expect a higher return for bearing risk, it is natural that insurance will have to cost more to avoid that risk. For each 2 per cent increase in risk premium, the expected cost of insurance will increase by 0.5 per cent.

Finally, it has often been observed (not entirely accurately) that portfolio insurance is a "buy high, sell low" strategy. It *is* a strategy that involves buying at the margin after market rises and selling after market declines. That does not necessarily mean "buy high, sell low," because a market increase or market decrease is not meaningfully correlated with *subsequent* movements in the market. Nonetheless, an increase in market volatility makes portfolio insurance more subject to "whipsaws." As the market drops, for example, portfolio insurance sells; when the market rebounds, portfolio insurance buys. This pattern will be more common, more pronounced and more costly in an environment of increased market volatility.

Table VIII clearly demonstrates this cost. The arithmetic mean cost of portfolio insurance increases from 1.3 to 3.0 per cent with an increase in market volatility from 13 to 28 per cent. The effect of increased market volatility on the median protected return is particularly striking; that falls all the way from 12.1 to 1.3 per cent. Interestingly, the geometric mean cost first increases with an increase in volatility, then decreases at higher levels. This results primarily from the substantial disparity between the standard deviations of the return distributions; the smaller standard deviation for the insured portfolio more than offsets the implied cost of the put option.

Table VIII Effect of Market Volatility on Return Characteristics of Insured Portfolios

	Annualized Standard Deviation of Equity Results							
	13%		*18%*		*23%*		*28%*	
	Uninsured	*Insured*	*Uninsured*	*Insured*	*Uninsured*	*Insured*	*Uninsured*	*Insured*
Mean Return (%)	15.0	13.7	15.0	13.0	15.0	12.4	15.0	12.0
Median Return (%)	14.3	12.1	13.6	9.0	12.8	5.4	11.8	1.3
Geo. Mean Ret. (%)	14.3	13.1	13.6	12.1	12.7	11.2	11.5	10.5
Avg. Beta	1.00	0.86	1.00	0.75	1.00	0.67	1.00	0.60
St. Dev. (%)	13.0	11.5	18.0	14.2	23.0	16.3	28.0	18.2
Skewness	0.34	0.80	0.47	1.24	0.61	1.64	0.74	2.01
Loss of Upside Capture (%)	0.0	1.9	0.0	4.1	0.0	7.0	0.0	10.3
Arith. Mean Cost (%)	0.0	1.3	0.0	2.0	0.0	2.6	0.0	3.0
Geo. Mean Cost (%)	0.0	1.2	0.0	1.5	0.0	1.5	0.0	1.0
Probability of Return Less Than:								
0%	11.9	0.0	20.7	0.0	27.3	0.0	32.2	0.0
Riskless Rate	30.9	37.0	37.3	47.6	41.4	55.0	44.4	60.5

Assumptions

Dividend Yield	= 3.0%	Percentage of Portfolio Insured	= 100%
Expected Market Return	= 15.0%	Insured Floor Return	= 0%
Riskless Rate	= 8.0%	Time Horizon	= 1 Year

The Role of Transaction Costs

Lastly, but perhaps most importantly, the investor must watch transaction costs; carelessness on this score translates directly into reduced rewards for a portfolio insurance strategy. Portfolio insurance is a high-turnover strategy; it can introduce from 40 to 100 per cent annualized turnover of the insured assets. If these assets are actively managed, they will already be experiencing perhaps 50 per cent turnover. Every 100-basis-point reduction in transaction costs translates into a 40 to 100 basis-point increase in the long-term return of the portfolio. For a $1 billion portfolio, with compounding, a single basis point per annum will be worth well over $2 million after a decade.

One way to look at the impact of increased transaction costs is to treat transaction costs as affecting the purchase price of the option. As Table IX shows, each increase in transaction costs on insurance trades has the same effect as buying an option at an "unfair" price. A 50-basis-point round-trip transaction cost has approximately the same impact as buying an option for 5 per cent above fair value. The relation is not linear. The larger the transaction cost, the larger the effective option mispricing; as the effective transaction costs rise, however, they force still greater turnover, leading to an accelerating increase in the magnitude of the effective option mispricing.

This mispricing can have two effects. It can lead to the possibility of "missing the floor." If an overpriced put option is purchased to protect against downside risk, the effect can be a slight shortfall vis-a-vis the intended floor return. It should be noted, however, that these shortfalls are modest and should not be viewed with alarm. The more pronounced impact is on the portfolio's mean and median returns. The average return on a protected portfolio, with an overpriced option, drops markedly vis-a-vis the average return without transaction costs. If transaction costs are zero, the arithmetic mean cost of a conventional portfolio insurance strategy is about 2.0 per cent. If transaction costs are 50 basis points, the cost of the portfolio insurance rises to 2.4 per cent.

Until recently, this 50-basis-point cost was on the high side for those who advocate the use of futures for portfolio insurance strategies. With the increased activity in portfolio insurance strategies, it is conceivable that the round-trip cost on some trades could rise to 100 basis points, or more.[14] If that were to happen, the cost of portfolio insurance would rise sharply.

The final example in Table IX is based on a 200-basis-point round-trip transaction cost. It is

Table IX Effect of Option Mispricing (or Transaction Costs) on Return Characteristics of Insured Portfolios

		Average Transaction Costs on Synthetic Put Trades					
		−50bp	*0bp*	*50bp*	*70bp*	*100bp*	*200bp*
		Equivalent Option Mispricing Percentage					
	Uninsured	*−5%*	*0%*	*+5%*	*+10%*	*+15%*	*+30%*
Mean Return (%)	15.0	13.4	13.0	12.6	12.1	11.7	10.3
Median Return (%)	13.6	9.5	9.0	8.6	8.1	7.6	6.1
Geo. Mean Return (%)	13.6	12.8	12.1	11.7	11.2	10.8	9.5
Avg. Beta	1.00	0.76	0.75	0.75	0.74	0.73	0.71
St. Dev. (%)	18.0	14.3	14.2	14.1	13.9	13.8	13.4
Skewness	0.47	1.23	1.24	1.25	1.27	1.28	1.33
Loss of Upside Capture (%)	0.0	3.9	4.1	4.4	4.6	4.8	5.4
Arith. Mean Cost (%)	0.0	1.6	2.0	2.4	2.9	3.3	4.7
Geo. Mean Cost (%)	0.0	0.8	1.5	1.9	2.4	2.8	4.1
Probability of Return Less Than:							
0%	20.7	0.0	0.0	29.8*	30.8*	31.8*	35.2*
Riskless Rate	37.3	46.5	47.6	48.6	49.7	50.8	54.5

Assumptions

Dividend Yield	= 3.0%	Insured Floor Return	= 0%
Expected Market Return	= 15.0%	Percentage of Portfolio Insured	= 100%
Market Volatility	= 18.0%	Time Horizon	= 1 Year
Riskless Rate	= 8.0%		

* Floor can be missed by 0.20, 0.40, 0.60 and 1.19 per cent, respectively.

not unreasonable to expect such a figure for portfolio insurance implemented *without* futures. Here the ultimate cost is enormous: Instead of portfolio insurance depressing long-term performance by 200 basis points per annum, the long-term performance drops by *470* basis points per annum. Even more alarming, the median return actually falls below the risk-free rate.

Table IX clearly illustrates the enormous importance of transaction costs in effective portfolio insurance management. If the manager of an insurance strategy is sloppy in effecting futures trades, or if the futures markets systematically present adverse mispricing for portfolio insurance strategies, the cost of insurance will rise sharply.

Implications

Portfolio insurance can provide an effective means of reducing the downside risk of a portfolio of risky assets. But this protection is not free; the investor must expect to "pay" in the form of reduced returns on the insured portfolio during up markets. While the investor cannot eliminate this cost, he does have some control over it.

First, the investor should examine the need for portfolio insurance, giving special attention to the insured floor return. Is a negative return—e.g., a loss of 10 to 20 per cent—tolerable? The floor return should be adjusted to reflect the investor's maximum tolerance for loss. The lower the floor return, the greater the reduction in the long-term cost of insurance.

Second, the investor can choose to insure only part of the assets in the portfolio. This will reduce the cost of insurance. The risk here, however, is that the entire portfolio will suffer increased losses in a disastrous market.

Third, the investor can extend the portfolio insurance horizon. Of course, if the investor is subject to annual review—by, say, the board of directors of a pension fund—annual insurance may be a political necessity. If this is not the case, however, he may opt for a multiyear horizon, which will sharply reduce the long-run cost of the insurance.

Finally, the investor can increase the risk (as measured by beta) of the underlying portfolio. This will increase the arithmetic mean of the insured portfolio, but it will also increase the probability of returns falling below the riskless rate. ■

Footnotes

1. See R. Ferguson, "How to Beat the S&P 500 (without losing sleep)," *Financial Analysts Journal*, March/April 1986. See also D. Luskin, "The Surprising Performance of Portfolio Insurance, Historical Evidence: 1883 to 1985" (Jeffries and Company, Inc., 1985).
2. A recent exception to this is a study by C.B. Garcia and F.J. Gould, "The Risks of Portfolio Insurance—Facts and Fallacies" (Working Paper, Graduate School of Business, University of Chicago, August 1986).
3. See R. Bookstaber and R. Clarke, "An Algorithm to Calculate the Return Distribution of Portfolios with Option Positions," *Management Science*, April 1983. We adapted the algorithm for the assumption that the underlying return distribution for the risky asset is lognormally distributed. Though this may not be a totally accurate assumption in practice, the algorithm is a means to illustrate the general impact of portfolio insurance parameters. Actual portfolios may deviate from this distribution, but the qualitative results will generally not change.
4. See H. Leland, "Who Should Buy Portfolio Insurance?" *Journal of Finance*, May 1980. See also M. Brennan and R. Solanki, "Optional Portfolio Insurance" (Working Paper No. 666, Faculty of Commerce and Business Administration, University of British Columbia, 1979). Brennan and Solanki go farther in attempting to evaluate some of the tradeoffs in an insured portfolio by assuming a lognormal return distribution and more specifics about the market's utility function.
5. For complete protection, the exercise price on the put option must be set at

$$E = (1+r^*)(S+P) - D,$$

where r^* is the insured floor return, S is the current stock price, P is the price of the put option with exercise price E, and D is the dividend on the stock before option expiration.

6. For a discussion of other ways to create portfolio insurance, see M. Rubinstein, "Alternative Paths to Portfolio Insurance," *Financial Analysts Journal*, July/August 1985.
7. The dynamic approach to creating portfolio insurance is discussed by M. Rubinstein and H. Leland in "Replicating Options with Positions in Stock and Cash," *Financial Analysts Journal*, July/August 1981.
8. See F. Black and J. Jones, "Simplifying Portfolio Insurance" (Goldman Sachs Research, August 1986).
9. The assumed risk premium of 7 per cent, equity volatility of 18 per cent per annum and dividend

Footnotes concluded on page 66.