RA Capital Market Expectations Methodology

Introduction

Since 2014, we have made public our capital market expectations through our interactive website. In the past, we also provided a set of asset class–specific methodology documents to describe our modeling approach. In 2021, we performed a holistic review of our models[1] and based on feedback from clients, we are replacing those individual documents with one omnibus document aimed at succinctly explaining the models we use to create our expectations.

A common saying is that "simplicity is the ultimate form of sophistication." In developing capital market expectations for global markets, it can be easy to fall down the rabbit hole of complexity. We strive to make our models comprehensible, robust, and able to deliver value in an out-of-sample fashion. Our approach is then guided by the three following principles:

1. avoid complexity whenever possible,
2. ensure model consistency across markets, and
3. capture intuitive macroeconomic linkages to markets.

Our models are not simply a combination of trailing returns. Economies do change over the long run—for instance, due to aging populations and changes in productivity. Utilizing an approach with strong economic foundations ensures those changes are translated into asset price forecasts.

This document does not aim to provide all of the background justification and empirical evidence supporting modeling choices. Instead, the aim is to focus on the meaningful aspects of our models such that the reader is able to understand our motivations, and if desired, could reasonably replicate the models. In addition, for those interested in a deeper dive into some of the subjects our models incorporate, we provide an additional reading section at the end of this document that highlights relevant articles.

Long-Horizon Forecasting

Most long-horizon forecasts start from the foundation of yield at the time of purchase as a key predictor of future returns. As a simple example, a default-free, zero-coupon bond held to maturity will have a full-life return exactly equal to the yield at the time of acquisition.

[1] Past users of our interactive site may notice some differences in our CME models as well as the display of the information. Although we have made some changes in an effort to streamline our models, the main drivers of return remain largely the same.

A natural extension from the starting-yield model is the Gordon Growth Model (GGM). The GGM states that the one-period stock return is based on the dividend yield at the end of the period and the subsequent growth in prices that come from growing cash flows. Because the GGM assumes price changes are directly related to changes in cash flows, it is therefore a constant-yield model,

$$r_{t,t+1} = \frac{D_{t+1}}{P_t} + g$$

In the preceding equation *r* represents the return from time *t* to time *t+1*, *D* are the dividends received at time *t+1*, *P* is the current price, and *g* is the dividend growth rate.

In our models, we further extend the yield + growth model to allow for the fact that asset yields can change over time (e.g., P/E multiples do expand and contract over time). Assets that are currently priced rich should be expected to return to fairer valuations in the future (i.e., negative return) and vice versa for assets currently priced cheap. We use the following simple decomposition of returns to generate the expected return equation (*DY* represents dividend yield):

$$r_{t,t+1} = \frac{D_{t+1}}{P_t} + \frac{P_{t+1}}{P_t} - 1$$

Because $1 = \frac{D_{t+1}}{D_{t+1}} / \frac{D_t}{D_t}$, we can multiply and expand the return equation as follows:

$$1 + r_{t,t+1} = \frac{D_{t+1}}{P_t} + \left[\frac{P_{t+1}}{P_t} \times \frac{\frac{D_{t+1}}{D_{t+1}}}{\frac{D_t}{D_t}} \right]$$

$$1 + r_{t,t+1} = \frac{D_{t+1}}{P_t} + \left[\frac{\frac{P_{t+1}}{D_{t+1}}}{\frac{P_t}{D_t}} \times \frac{D_{t+1}}{D_t} \right]$$

$$1 + r_{t,t+1} = \frac{D_{t+1}}{P_t} + [(1 + \Delta Valuation) \times (1 + \mathrm{g})]$$

$$1 + r_{t,t+1} = \frac{D_{t+1}}{P_t} + 1 + g + \Delta Valuation + \Delta Valuation \times g$$

Given that $\Delta Valuation \times g$ is generally a very small number, we can approximate and simplify the above equation as follows:

$$1 + r_{t,t+1} \cong \frac{D_{t+1}}{P_t} + 1 + g + \Delta Valuation$$

by subtracting 1 from both sides of the equation and shortening the notation,

$$r_{t,t+1} \cong DY + g + \Delta V$$

The expected returns depicted in this document are in real terms unless otherwise specified. In addition, the Research Affiliates Asset Allocation Interactive (AAI) site offers two models of expected return: 1) yield + growth and 2) valuation dependent. The yield + growth model is inspired by the GGM and includes starting asset-class yields plus long-term trend growth rates. The valuation-dependent model includes changes from shifts in valuation. AAI offers two models to compensate for the inherent subjectiveness in modeling fair values for asset classes. We understand that AAI users may have different assumptions around fair value. Therefore, we seek to serve a broad audience by offering users a choice of model.

Both models have a horizon of 10 years, consistent with Ilmanen et al. (2019): "For multi-decade forecast horizons, the impact of starting yields is diluted, so theory and long-term historical average returns (or yields) may matter more in forecasting expected returns." All returns displayed on AAI are annualized geometric averages.

Why Long Term, not Short Term?

In contrast to the many studies that focus on shorter horizons (1 year, 1 month, or even less) to forecast asset-class returns, we adopt a 10-year horizon. Long-horizon forecasts are able to focus on the central tendencies of asset returns while ignoring the transitory influences that plague short-horizon signals. For this reason, some consider estimating long-horizon returns as being easier than estimating short-horizon returns, but we find that view to be an oversimplified and somewhat misleading interpretation. After all, the error in most forecasts increases with the time horizon.

Long-horizon forecasting of valuations is aided by the tendency[2] of most asset classes to mean revert to a "fair value." The challenge then becomes finding the future (mean) fair value with the limited data available for running the necessary statistical tests. We have two choices: 1) use the data that are available and estimate the future fair value from the average of the existing data or 2) assume future regime shifts such that the future fair value will differ from the past. Because we are estimating models using fundamental metrics, we choose the former approach, simple models with minimal moving parts.[3] We

[2] In long-horizon forecasts, historical returns are not good measures of central tendencies of asset returns, although historical returns may be suitable for short-term forecasting of stationary time series.
[3] Other walk-forward methods, including simulation and trees, are means to extract distributions of outcomes including tail risk, but are not appropriate for use with fundamental-based models.

provide further rationale for this decision in the discussion of accuracy versus precision later in this document.

To the extent that one may desire to incorporate long-run regime shifts in the process, we believe the best approach is to gather those expectations separately. A survey is one method to acquire a holistic sampling of the investment landscape.[4] In contrast, relying on historical simulations to identify long-run regime shifts may lead to data overfitting, bias, and unintended out-of-sample results. The history of structural economic changes is limited by definition, which warrants strong caution in the use of data-mining techniques for the purpose of long-term forecasting.

Any forecast has variation around it. Some of the variation arises from variance in the model itself, which we refer to as model uncertainty: for instance, uncertainty about the parameters of the model or even uncertainty about the choice of the model. In practice, we are concerned about how well or poorly a model identifies the correct average expected returns. Second, some of the variation arises from asset price volatility, or risk (i.e., actual returns may not hit the average expected return in any given 10-year period). We believe models based on moving averages of historical data helps reduce model uncertainty compared to models based on complex relationships, which are prone to overfitting noisy data. In other words, simpler models may coincide with a consistent bias across models, which would have a smaller impact on our cross-sectional predictions than more complex asset-specific models.

Finally, in many cases determining the success/value of individual long-horizon expectations is difficult because of a lack of historical data. This limitation affects both time-series comparisons as well as cross-sectional comparisons. For this reason, we opt for portfolio tests to measure the value of our long-horizon forecasts, consistent with the goal of developing strategic asset allocations. The benchmark of success then becomes: Over the past decades, if we invested based on these expectations (e.g., overweight assets with higher expected returns, and vice versa), would we have outperformed a simple equal-weight blend of assets?

Objectives of Capital Market Expectations

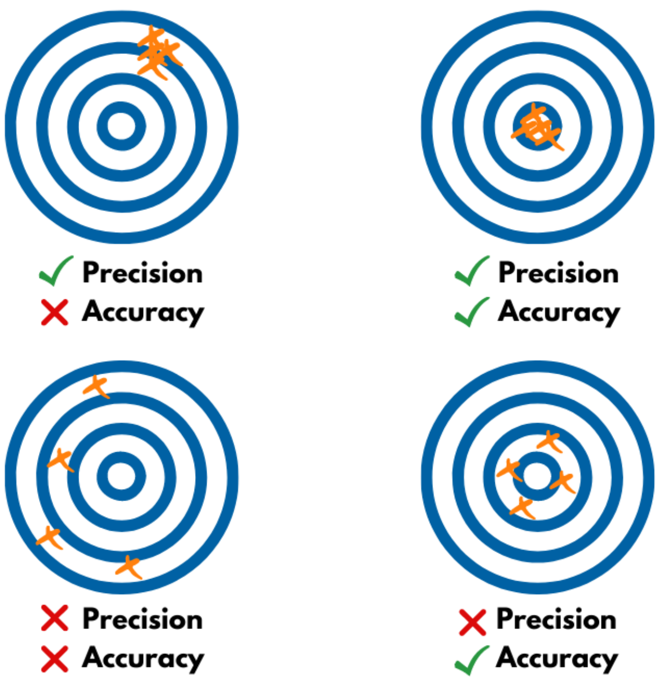
Before moving on to a description of our models, we believe a discussion of objectives is important because they motivate our modeling decisions. Our primary objective in developing our capital market expectations (CMEs), also known as long-horizon expected returns, is the need to generate strategic asset allocation portfolios. Whereas readers of this document may have different motivations, defining long-horizon expectations is a need shared by many of the users of the data we provide through AAI.

[4] Comparing our models to investment community expectations could be a future differentiating feature of AAI. Even a fraction of AAI users' completing a simple survey on expected returns would equate to a few thousand responses.

Another common use of the data is to estimate the absolute magnitude of a particular return forecast (i.e., the specific return asset X is expected to earn over the next 10 years). We believe this objective is far less useful for investors. Thus, we are doggedly interested in, and focused on, the system of returns and much less concerned with any individual return. For us, the forest is what matters, not the trees.

We can also support the reason for our focus through the lens of accuracy and precision. In an ideal world, the optimal outcome would be to achieve both high accuracy and precision. In the real world, this is a tremendous challenge.

On the one hand, in constructing portfolios, being precise across the system of assets is much more important than being accurate for only a subset of assets. Biasing expected returns up or down will not affect the resultant portfolio as long as the bias is equally reflected in all asset class expectations. On the other hand, having one or two very accurate expectations, while others are scattered about will generate biased strategic portfolio allocations. The following simple graphic illustrates the trade-off between accuracy and precision:



The implications of this distinction are relevant in multiple areas. The first is trading. If CMEs are both accurate and precise in both magnitude and horizon, there is no reason to trade more frequently than the signal horizon. Otherwise, the investor would simply be throwing away the accuracy of the forecasts. The second is that discussions around the accuracy/precision of specific equity expected returns are mostly unhelpful in achieving asset allocation objectives, and even run the risk of introducing inconsistencies to the process and degrading the resultant portfolio allocations. The right question to ask is “Are equity returns biased in relation to the expectations of other assets?”

Our objective, however, is not to eliminate bias (impossible), but to the greatest extent possible ensure the presence of bias is consistent across asset classes. We employ two methods to accomplish this: 1) we use simple, straightforward models to estimate far into the future and 2) we introduce foundational drivers of return (e.g., unemployment rate) with respect to all assets.

We recognize that some asset owners, in response to funding requirements and operational considerations, need to be very focused on accuracy and the absolute level of specific expectations. Although we strive to represent accurate estimates of the absolute value of future returns, we do not seek to improve a particular asset class return at the risk of introducing bias into the system.

Capital Market Expectations Models: The Details

Real Gross Domestic Product

Gross domestic product (GDP) measures the production or output of goods and services within a country during a certain period. We focus on real GDP (RGDP), which excludes changes in the value of output merely due to changes in price level.

Modeling Framework

By employing a simple decomposition, we recognize two key drivers of real production: productivity and population. Productivity measures a country's efficiency in producing goods and services, whereas population captures the potential number of people who can contribute to the production. Our emphasis on productivity and demographic trends builds on our firm's multi-year work on these topics, as exemplified by Chaves and Arnott (2012), who brought to light a significant connection between capital markets, growth, and demography. For each country, we approximate productivity by the level of real output per number of people,

$$RGDP = \frac{RGDP}{Population} \times Population$$

Hence, a 10-year real GDP growth forecast can be approximated as the sum of expected output-per-capita growth and expected population growth,[5]

$$E[RGDP\ Growth] \cong E\left[\frac{RGDP}{Population}\ Growth\right] +\ E[Population\ Growth]$$

To predict population growth, we rely on data from the United Nations (UN) Population Database,[6] which provides population predictions for multiple decades in the future. The data are lagged by one quarter. To predict output-per-capita growth, we employ the following three driving forces:

$$E[\frac{RGDP}{Population}\ Growth] = E[Normal\ Growth] +\ E[Demographic\ Effect] + Adjustment$$

We estimate *Normal Growth* by computing an exponentially weighted moving average (EWMA) of past quarterly rates of output-per-capita growth; the half-life term is set to five years.

The *Demographic Effect* is inspired by the research of Chaves and Arnott (2012) on the impact of structural demographic changes on the economy and capital markets, and we define it as

$$Demographic\ Effect =\ S(MY)$$

[5] To appreciate the approximation, denote the growth rate of $RGDP$ between any two periods by g, where for instance $RGDP_1 = RGDP_0(1+g)$. Similarly, let y define the growth rate of $RGDP/Population$ and z the growth rate of $Population$. By definition, the following is true:

$$RGDP_1 = RGDP_0(1+g) = \frac{RGDP_0}{Pop_0}(1+y) \times Pop_0 \times (1+z)$$

simplifying the terms,

$$1 + g = (1+y)(1+z) = 1 + y + z + y \times z$$

Finally, since $y \times z$ is generally a very small number, the growth rate of $RGDP$ can be approximated as

$$g \cong y + z$$

[6] https://population.un.org/wpp/Download/Standard/Population/

where MY is a weighted average of the middle-aged-to-young ratio expected over the subsequent 10 years as derived from the UN data.[7] The function $S(MY)$ is defined as

$$S(MY) = -\left[\frac{1}{\left(1 + e^{-4 \times (MY-1)}\right)} - \frac{1}{2}\right] / 100$$

$S(MY)$ is a sigmoid function that maps a country's demographic structure to potential growth tail or headwinds.

Lastly, *Adjustment* adjusts for the negative skew of the distribution of output growth rates. Note that output growth is usually characterized by relatively long periods of moderate growth and relatively short periods of contraction. The skewness of productivity growth implies that, during and following a recession, the EWMA may lead to particularly pessimistic forecasts. Hence, the *Adjustment* accounts for the recession-related dynamic by introducing an adjustment as

$$Adjustment_t = \frac{1}{N}\sum_{i=1}^{N} Median\ Past\ Growth_{i,t} - Average\ Past\ Growth_{i,t}$$

where i indicates a country, t a month, and the median and average statistics are computed over a 10-year period for output-per-capita growth. We compute this adjustment separately for emerging and developed countries. This adjustment is significant in magnitude—about 40–50 basis points—during recessions and it gradually shrinks to zero during expansions.

[7] To compute the expected *MY* ratio, we leverage on Arnott and Chaves's (2012, *FAJ*) research and calculate it as a weighted average across relevant demographic shares. The respective demographic shares are listed in the following table together with the corresponding weights. The weights are based on historical relationships.

Weights for Young Group						Weights for Middle Group			
15–19	**20–24**	**25–29**	**30–34**	**35–39**	**40–44**	**45–49**	**50–54**	**55–59**	**60–64**
4.93%	15.42%	21.56%	23.36%	20.81%	13.93%	0.00%	12.53%	31.92%	55.54%

Formally, the total population in the Young and Middle groups is a weighted average with the relative weights (w) listed in the preceding table,

$$Middle_{j,t} = \textstyle\sum_{k=1}^{K} w_{i,k} E_t\left[Population_{i,t+120}^{k}\right] \text{ and } Young_{j,t} = \textstyle\sum_{k=1}^{K} w_{i,k} E_t\left[Population_{i,t+120}^{k}\right]$$

where k represents a demographic share and $K = 4$ for the Middle group and $K = 6$ for the Young group. The expected population levels for the different share groups 10 years in the future are from the United Nations Population Database. Lastly, the *MY* ratio is defined as

$$MY\ Ratio = Middle/Young$$

Inflation

We model a country's headline (All Items) not-seasonally adjusted inflation index. Inflation captures the change in the price level across goods and services within a country over a period of time. Unless specified otherwise, we work with year-over-year inflation, defined as

$$Inflation_{t+12} = \frac{Consumer\ Price\ Index_{t+12}}{Consumer\ Price\ Index_{t}} - 1$$

where t indicates a month. The year-over-year computation eliminates seasonal fluctuations of the index. Inflation data are generally released at a monthly frequency with a few exceptions, such as in Australia. The data are also released with a lag, usually of a few weeks, such that in modeling future inflation from a particular date, we only use data available as of that date.

Our modeling approach builds on the recognition of the distinct role long-term inflation trends play versus cyclical fluctuations. This distinction is key in the inflation-forecasting literature, for instance, Faust and Wright (2013), as well as in the bond-forecasting literature, in particular, Cieslak and Povala (2015).

Modeling Framework

We compute *Long-Term Inflation*, expected inflation in 10 years' time, by employing EWMAs of past *core* inflation rates. The calibration of the EWMA for developed economies is based on a 10-year window of data and a half-life of five years, whereas for emerging markets the half-life parameter is set to two years to account for greater volatility. Similar to the model for GDP expectations, we make a further adjustment to the long-term inflation rate to address skew in the inflation data.

From there we compute an implied path of inflation at the speed of convergence of current inflation to its long-term "fair" value, speed that can be approximated at about 3% a month (i.e., 3% of the remaining gap is closed every month). Average inflation over the next decade can be approximated by

$$\begin{aligned} &Inflation\ Forecast \\ &\qquad = 30\% \times Current\ Headline\ Inflation + 70\% \times Long\ Term\ Inflation \\ &\qquad + Adjustment \end{aligned}$$

Consistent with the literature, we employ core inflation to determine long-term trends because the series displays less volatility and, therefore, better captures the underlying relevant dynamics. A core Consumer Price Index (CPI) is generally defined by excluding energy and food indices from the headline basket, although country-specific definitions may slightly vary. If core CPI is not available for a country, then long-term inflation is computed by calculating the EWMA of past headline inflation rates.

Lastly, *Adjustment* adjusts for the skew in the distribution of inflation rates. The skewness of inflation implies that, during and following inflation peaks, the EWMA may lead to particularly high forecasts. Hence, the *Adjustment* accounts for this dynamic as

$$Adjustment_t = Median\ Past\ Inflation_t - Average\ Past\ Inflation_t$$

where *t* indicates a month, and the median and average statistics are computed over a 10-year period. Unlike output, we calculate a separate adjustment for each country, because country inflation rates can be contemporaneously very different, especially in emerging markets.

Government (Treasury) Bills

Treasury bills are a type of government debt with a maturity less than one year that do not pay any interim cash flows. Treasury bills are often used to define the risk-free rate in a particular country, because of their short duration and because a country that owns its currency should always be able to print to meet short-term funding needs (although this is not always the case).

For our purposes, let $TBill$ indicate a generic Treasury bill security of three-month maturity. In a nutshell, our modeling framework is based on three components: 1) a macroeconomic-based expectation of the "neutral" or "equilibrium" value of the $TBill$ rate in the long run, 2) inflation expectations, and 3) a lower bound to its value to account for the limits of conventional monetary policy. Our inflation expectations are described in the section on inflation.

The Fisher equation states that the nominal risk-free rate is the sum of the real interest rate and expected inflation. Our model highlights an association between real output growth and the equilibrium real rate of interest; indeed, faster-growing economies should deliver on average higher rates of return. In addition to being intuitive, this association is motivated by a growing body of literature (see, for instance, Laubach and Williams' [2003] seminal work on the topic as well as our own research summarized by Garg and Mazzoleni [2017]).

Modeling Framework

Similar to inflation, we model a path of T-bill rates from the current level to the equilibrium level at a rate of 3% a month. As with inflation, average T-bill rates over the next decade can be approximated as the following:

$$Expected\ TBill\ Rate = 30\% \times Current\ TBill + 70\% \times Equilibrium\ TBill\ Rate$$

The $Equilibrium\ TBill\ Rate$ is a function of a macroeconomic model and a lower bound. We first start by defining a long-run estimate of T-bill rates based on a simple macroeconomic model,

$$TBill_{LR} = Country\ Factor + RGDP\ Growth\ Forecast + \ Inflation\ Forecast$$

Earlier in the document, we explained the methodology for our forecasts of RGDP growth and inflation. The $Country\ Factor$ is used to approximate the different liquidity premia of T-bill-equivalent markets across countries and is estimated as the empirical difference between cash rates, real GDP growth, and inflation.

To estimate the country factor, we proceed in three steps. First, we construct historical series of $TBill - RGDP\ Growth\ Forecast - LongTerm\ Inflation$ and employ an EWMA to estimate the average value, using a window of 10 years and a half-life of five years. Second, to minimize the risk of noisy estimates, we take the median of the values found in step one across developed or emerging markets; hence, the $Country\ Factor$ for a developed market is the median EWMA across all developed markets. Third, to avoid outliers or unintuitive results, we employ our priors and also set an upper (0.1) and a lower bound (−0.1) for the final value of the $Country\ Factor$ in order to ensure it stays within a reasonable range.

Our economic prior dictates that, over the long run, cash rates move in lockstep with growth and inflation. Instead, the $Country\ Factor$ may reflect a variety of forces, such as safety and liquidity premia, and therefore, we continuously update its estimates via an EWMA.

Lastly, we set a lower bound to nominal interest rates, which we estimate to be −0.75% (a number informed by the experience of some European countries). Hence,

$$Equilibrium\ TBill\ Rate = \max\,(-0.75\%, TBill_{LR})$$

Asset Allocation Interactive (AAI) Representation

The following table shows how the methodology is depicted in both the yield + growth and valuation-dependent pricing model breakdowns on our AAI website:

Treasury Bills (Cash Rates)

		Nominal	Real
Current Yield	T-Bill Yield	$Current\ (Nominal)\ TBill\ Rate$	
Growth	Inflation	N/A	$-Inflation\ Forecast$
Valuation	Change in T-Bill Yield	$Expected\ (Nominal)\ TBill\ Rate - Current\ (Nominal)\ TBill\ Rate$	

Bonds

A bond is a security for which a borrower promises to pay the principal of a loan to the lender at some time in the future and typically to pay periodic interest payments over the life of the bond. Due to this structure, a bond can have both interest rate (duration) and credit (default) risk.

We model developed-market government bonds (assumed free of default risk), inflation-indexed bonds (TIPS, also default-risk free), emerging market bonds (local currency), corporate bonds (investment grade and high yield), hard-currency emerging-market bonds, and bank loans. At a high level, our approach builds on three return drivers: the nominal T-bill yield and its expected future yields; the risk premium demanded by investors to hold a certain bond; and potential credit losses. The nominal T-bill rate is influenced by real output growth and inflation, which we discussed in the preceding section. Hence, the remainder of this section will focus on the risk premium and credit loss return drivers.

Government bond yields are often succinctly characterized by three factors: level, slope, and curvature. We could theoretically map our model formulation to these factors. Originally, Litterman and Scheinkman (1991) estimated these drivers via a principal component analysis of the term structure of yields. Therefore, by construction, they are simply a *linear combination* of different yields on the yield curve, whose exact combination can vary over time. Hence, it would be straightforward to frame our long-term expectations in terms of these three yield-curve factors. For instance, the level factor of the US yield curve could be approximated as the average of the yields across all maturities, the slope factor as the yield spread between long- and short-term yields, and the curvature as a combination of the T-bill yield, 2-year yield, and 10-year yield.

Importantly, we do not rely on a purely statistical model, limited to the three drivers of level, slope, and curvature, to predict yields. We also introduce macro drivers, consistent with the most recent academic literature.

Modeling Framework

The return on a bond comprises up to four components, depending on the riskiness of the security. The following equation is the general formulation:

$$Bond\ Return = Yield + Roll\ Return - Credit\ Loss + Bond\ Valuation\ Return$$

This model is motivated by a first-order Taylor approximation of bond price dynamics. In a first-order Taylor approximation, if a bond yield rises by one percentage point, then its price falls by an amount approximately equal to its duration/100.[8] We opt for a first-order approximation rather than a second order, which would have included a convexity term in the preceding expression.[9] To set the notation, we generalize the preceding equation by expressing the returns of a bond having maturity τ over a one-year period as follows:

$$E_t[R_{t+1}^{\tau}] = y_t^{\tau} - D \times E_t[y_{t+1}^{\tau-1} - y_t^{\tau}] - E_t[Credit\ Losses_{t+1}]$$

[8] A first-order Taylor approximation of a generic function $f(x)$ about x_0 is defined as $f(x) \cong f(x_0) + f'(x_0)(x - x_0)$. Hence, a first-order approximation of a bond price about price P_0 and yield y_o is given by $P \cong P_0 + \frac{dP}{dy}(y - y_0)$, from which follows that

$$Bond\ Price\ Return = \frac{P - P_0}{P_o} \cong \frac{1}{P_0}\frac{dP}{dy}(y - y_0) = -D \times (y - y_0)$$

Lastly, bond (total) returns are simply the sum of bond price returns and the bond yield.

[9] We ignore convexity for two reasons: first, its effect tends to be very small; second, convexity data at an asset class/index level is difficult to obtain. Even with duration, data limitations force us to utilize a single value instead of looking at key rate durations for the basket of securities in the index (If we included mortgages in this analysis, then convexity would be a bigger issue).

The yield y_t^τ of a fixed-income security or portfolio is the interest rate that ties the stream of future cash flows to the current price. The term D indicates the duration of the security and $E_t[Credit\ Losses_{t+1}]$ indicates our expectation of net losses due to outright default as well as transition. Lastly, the notation $E_t[R]$ indicates the expectation of R taken at time *t*.

We distinguish between roll yield and valuation changes. To do so, we add and subtract $y_t^{\tau-1}$ from the preceding equation to rewrite it as

$$E_t\left[R_{t,t+1}^\tau\right] = y_t^\tau - D \times E_t[y_{t+1}^{\tau-1} - y_t^{\tau-1} + y_t^{\tau-1} - y_t^\tau] - E_t[Credit\ Losses_{t+1}]$$

With a simple rearrangement of the terms, we can highlight the two components as follows:

$$E_t\left[R_{t,t+1}^\tau\right] = y_t^\tau \underbrace{-D \times (y_t^{\tau-1} - y_t^\tau)}_{Roll\ Return} - E_t[Credit\ Losses_{t+1}] \underbrace{-D \times E_t[y_{t+1}^{\tau-1} - y_t^{\tau-1}]}_{Bond\ Valuation\ Return}$$

Unlike equities and other long-life assets, different bonds mature at different horizons. Thus, we need to develop a process to model our 10-year expected return on an instrument that may have a maturity shorter than 10 years (e.g., 2-year US Treasury bond). Our approach is to focus on modeling a near-constant maturity exposure created by holding any bond for 1 year and then rolling that exposure back into a new bond issuance at the original maturity. Our 10-year return forecasts are then expressed as the average yearly returns over the horizon considered,

$$Expected\ Bond\ Returns = \frac{1}{10}\sum_{k=0}^{9} E_t\left[R_{t+k,t+k+1}^\tau\right]$$

summarized as the sum of the average yield, average roll yield, average credit loss, and average valuation component.

Based on the preceding framework, the key to modeling bonds is to determine the path of yields into the future based on three components: real T-bill rates, inflation, and risk premia. The *Expected Average Treasury Yield* over the next decade, $E_t\left[y_{t,t+10}^\tau\right]$, can be thought of as the average of expectations of those components,

$$E_t[y_{t\to t+10Y}^\tau] = \underbrace{\frac{1}{10}\sum_{k=0}^{9} E_t[r_{t+k}]}_{Real\ T-bill\ Rate} + \underbrace{\frac{1}{10}\sum_{k=0}^{9} E_t[\pi_{t+k}]}_{Inflation} + \underbrace{\frac{1}{10}\sum_{k=0}^{9} E_t[\phi_{t+k}^\tau]}_{Risk\ Premium}$$

where the sum of first two terms is the expected average nominal T-bill rate over a 10-year period, which we discussed in detail in the Treasury Bills section. Armed with our model, we then only need to add the risk premia expected in equilibrium yields and the speed of convergence toward this equilibrium (i.e., mean reversion of bond risk premia). In addition, if an asset is not default-risk free, then we also need to reflect expectations about credit losses, such as downgrades and defaults. Equipped with this general framework, we now explain the formulations for individual bond classes.

Modeling: Developed (Nominal) Government Bonds

The developed countries for which we estimate expected government bond returns are the United States, Canada, Australia, Hong Kong, Japan, United Kingdom, Germany, France, Italy, and Spain.

The modeling approach follows six steps:

1. The bond term (risk) premium ϕ of country j is measured using the slope of the country yield curve,

$$\phi_{j,t}^{\tau} = y_{j,t}^{\tau} - y_{j,t}^{0.25}$$

   and we estimate its fair value $\widehat{\phi_{j,t}^{\tau}}$ country by country by employing an EWMA (20-year half-life with a 50-year window or what the data availability allows).

2. We then estimate the speed of convergence, ρ, month by month by running the pooled regression (with a minimum of 5 years of data required),

$$\phi_{j,t+1}^{\tau} - \phi_{j,t}^{\tau} = \rho^{\tau} \times \underbrace{\left(\phi_{j,t}^{\tau} - \widehat{\phi_{j,t}^{\tau}}\right)}_{Gap\ from\ Fair\ Value} + \varepsilon_{j,t+1}^{\tau}$$

3. We impose the mean-reversion parameter to be bounded within −0.015 and −1.0 to ensure a minimum and a maximum level of yield mean reversion. We label the resulting coefficient as $\hat{\rho}_{t,B}^{\tau}$.[10]

4. To avoid “uneven” speeds of mean reversion across a bond curve, we refine the speed of mean reversion across all τs by fitting the following function across three points, where x is the number of months over 1, 10 and 30 years, and $\widehat{\rho}_{t,B}^{\tau}$ are the speed estimates from the pooled regression.

$$\widehat{\rho}_{t,B}^{\tau}(x) = \alpha + \beta_1 x + \beta_2 x^2 \text{ where } x = [12{,}120{,}360]$$

   Using this equation, we solve for β_1 and β_2 and use those estimates to calculate ρ^{τ} across all tenors of the yield curve. If $\widehat{\rho}^{1}$ is not available, we utilize $\widehat{\rho}^{5}$ (x=60). Again, to avoid unintuitive results, we bound the outcome of ρ^{τ} to be within −0.015 and −1.0.

5. We average the estimated ρ^{τ} over the previous 12 months to smooth potentially noisy estimates (the averaging takes place after fitting the curve). The resulting parameters are labeled $\rho_t^{\tau,*}$.

[10] Suppose the current slope is 2% and the fair value is estimated at 1%; this implies a 1% gap that is expected to disappear over time. A coefficient of −0.015 means that every month 1.5% of the gap will be closed, after 10 months about 15% of the gap will be closed, and after 10 years about 84% will be closed. A coefficient of −1.0 means that the entirety of the gap will be closed after one month (i.e., the slope jumps from 2% to 1% in one month).

6. Lastly, we employ the selected value of $\rho_t^{\tau,*}$ together with our cash-rate model to obtain the expected bond yield for any maturity at any month in the future.

Indeed, the Treasury bill model provides us with the expected path of the risk-free rate over time. Similarly, the model described by the preceding steps provides the expected path of the slope of the yield curve. By joining these two paths, we can obtain the expected dynamic over time of a bond yield and extrapolate its expected returns.

Modeling: Inflation-Indexed Bonds (TIPS)

Similar to nominal government bonds, we define the TIPS term premium of country j as

$$\phi_{j,t}^{\tau-1} = y_{j,t}^{\tau-1} - r_{j,t}^{\text{ref}}$$

where $y_{j,t}^{\tau-1}$ is the TIPS yield and $r_{j,t}^{\text{ref}}$ are the shortest reliable yields of the market real-yield curve in each country. Unlike government bonds that utilize the three-month yield, short-term real rates are particularly illiquid. Therefore we use the shortest liquid tenor.

In modeling TIPS and starting from market real yields, we quickly recognize a divergence between implied breakeven inflation rates (nominal yields – real yields) and our own forecast of inflation. We thus model a convergence of these two by assuming market real yields revert to our expectation of real yields, one decade in the future, using the 3%-a-month rule described earlier. In this way we maintain a linkage between market expectations of inflation and real yields and our own expectations of inflation and real yields.

Modeling: Emerging Markets Bonds (Local Currency)

The emerging market countries for which we estimate expected local-currency government bond returns are Brazil, India, Indonesia, Mexico, Poland, Russia, South Africa, South Korea, China, Philippines, Malaysia, and Colombia.

To model the term-premia dynamics of these bonds, we follow the same steps as for nominal government bonds. First, we define the bond term premium of country j as

$$\phi_{j,t}^{\tau-1} = y_{j,t}^{\tau-1} - y_{j,t}^{0.25}$$

where $y_{j,t}^{0.25}$ is the local short-term T-bill rate in the country of interest. We measure the fair value of the term premium country by country by employing an EWMA (20-year half-life with a 50-year window or what the data history allows) and estimate the speed of mean reversion via the following regression:

$$\phi_{j,t+1}^{\tau-1} - \phi_{i,t}^{\tau-1} = \rho \times \left(\phi_{j,t}^{\tau-1} - \widehat{\phi_{j,t}^{\tau-1}}\right) + \varepsilon_{j,t+1}^{\tau-1}$$

Note that when estimating the speed of convergence, we use all countries in our sample to increase the parameter robustness to potential noise in the data (i.e., we include both emerging and developed economies).[11] Lastly, we follow the same third, fourth, fifth, and six steps as for developed government bonds.

Although a rare phenomenon because a country can typically print its own currency, some emerging market nations have defaulted on their local-currency borrowings. Therefore, unlike for developed-market sovereign bonds, we estimate the potential credit losses for emerging market nations' bonds. As a general formulation, we define the expected average 10-year credit losses of a security with maturity τ and credit rating q as

$$E_t\left[Credit\ Losses_{t,t+10Y}^{\tau,q}\right] = Historical\ Average\ losses\ due\ to\ net\ default$$

Based on historical data, we assume the following: default rate = 0.18% and recovery rate = 40%. Unlike credit bonds, we do not explicitly consider downgrades with respect to emerging-market local debt because, for example, a credit-rating downgrade will not automatically trigger a sale event as it could with an investment-grade asset and instead assume a netting of upgrades and downgrades over time.

Modeling: Corporate Bonds and Hard-Currency EM Bonds

The risk premium for corporate bonds, both investment grade (IG) and high yield (HY) and hard-currency emerging market bonds, can be decomposed into two components: 1) the term premium of a duration-matched US Treasury bond and 2) the spread of the actual security with respect to the duration-matched US Treasury bond. More formally, let y_t^{DMGB} be the yield of the duration-matched government bond and y_t^{τ} the yield of the security for which we are estimating the expected return. We described the modeling of y_{t+k}^{DMGB} (and ϕ_{t+k}^{DMGB}) in the previous section dedicated to developed-market government bonds. Thus, the yield spread, $y_t^{\tau} - y_t^{DMGB}$, and its dynamics are the only remaining input needed as pertains to the valuation component of the yield.

To model the yield spread, $y_t^{\tau} - y_t^{DMGB}$, we follow the same steps that we did for nominal government bonds. First, let the premium be identified as

$$\theta_t^{\tau-1} = y_t^{\tau-1} - y_t^{DMGB}$$

where $y_t^{\tau-1}$ is the yield of a corporate or hard-currency bond and $y_t^{DMGB}$is the yield of a maturity-matched US government bond. We measure the fair value of the term premium by employing an EWMA (20-year half-life with a 50-year window or what the data history allows) and estimate the speeds of mean reversion and yield convergence. We then follow the same third, fourth, fifth, and sixth steps as described for government bonds.

Pertaining to the credit-loss term, $E_t\left[Credit\ Losses_{t,t+10Y}^{\tau,q}\right]$, we include expectations for both net defaults as well as net downgrades because the latter can result in realization of loss

[11] Our sample for emerging market bonds covers a shorter time period and is characterized by important idiosyncratic factors. Wary of the noise that may affect our estimates, we opt to also include developed economies in our estimations in an effort to reduce the impact of noisy observations.

if a bond is downgraded from one asset class to another (e.g., from investment grade to high yield),

$$E_t\left[Credit\ Losses_{t,t+10Y}^{\tau,q}\right] = Historical\ Average\ losses\ due\ to\ net\ defaults + Historical\ Average\ net\ downgrades \times Expected\ DTS$$

where DTS is *Duration Times Change in Spread* and the spread refers to the yield difference between high-yield and investment-grade bonds.[12]

	Default Rate	Recovery Rate	Transition Rate
Investment Grade	0.1%	70%	6%
High Yield	5.5%	40%	1%
EM Hard Currency	2.8%	55%	0%

Modeling: Bank Loans

A bank loan is a floating-rate asset with a duration close to zero and some specific features (e.g., yield floor).[13] To illustrate our modeling process for bank loans, assume the yield is given by y_t^{BL}. The loan has a duration of close to zero and the yield dynamics are based on those of short-term high-yield bonds with a BB rating, with a yield defined as $y_t^{0.25,BB}$. Specifically, the expected yield in 10 years is calculated as

$$E_t[y_{t+10}^{BL}] = \max\left(E_t[\tilde{y}_{t+10}^{BL}], \text{Bank Loan Floor}\right)$$

where E_t indicates the expectation at time t, and the real bank-loan floor is 1% (we obtain the nominal floor by adding our inflation forecasts). The general formulation of the term $\tilde{y}_{t+10}^{BL}$ is

$$E_t[\tilde{y}_{t+10}^{BL}] = \underbrace{y_t^{0.25,BL}}_{Current\ BL\ Yield} + \underbrace{E_t\left[\Delta y_{t+10}^{0.25,BB}\right]}_{\substack{Expected\ Change \\ in\ BB\ Yield}}$$

where the expected change in $\Delta y_{t;t+10}^{0.25,BB}$ is modeled as described in the Corporate Bonds section. Hence, if the yield on corporate bonds is expected to rise, the yield on bank loans will be expected to rise. To further appreciate how these inputs map to expected returns, recall our generic τ-bond one-period return decomposition,

[12] To approximate the potential losses due to a sale event triggered by a downgrade, we employ the expected spread between high-yield and investment-grade bonds from the United States.

[13] Bank loans have a few notable peculiarities. As regards credit losses, bank loans used to be considered relatively safe because they are collateralized and are higher in the capital structure. Now, some bank loans are "cov-lite" loans, which have few covenants, and many companies have a loan-only capital structure. In the latter case, the loan is still senior in the capital structure, but because the capital structure consists only of loans, they are senior to nothing. For this reason, default rates have recently risen. An additional consideration is that a bank loan's expected yield cannot fall below a certain threshold. Bank loans usually have a built-in floor, so although the rate on a loan floats, as Treasury rates fall, the bank loan rate is capped on the downside. Also a consideration is that bank loans have variable coupons (interest rates) and no call protection. The result is that their duration is not well defined. To compensate for this unknown, they are duration matched to short rates.

$$E_t\left[R_{t,t+1}^{\tau}\right] = y_t^{\tau} - \underbrace{D \times (y_t^{\tau-1} - y_t^{\tau})}_{Roll\ Return} - E_t[Credit\ Losses_{t+1}] - \underbrace{D \times E_t[y_{t+1}^{\tau-1} - y_t^{\tau-1}]}_{Bond\ Valuation\ Return}$$

In the case of bank loans, we assume a duration close to zero, which negates the components dependent on duration.

Lastly, we model credit losses as

$$E_t\left[Credit\ Losses_{t,t+10}^{\tau}\right] = \max\left(\text{Historical Losses BB Bonds}, 2.15\%\right)$$

We approximate the historical losses for bank loans using the historical loss measures of high-yield (BB) corporate bonds. We also include a floor on defaults (4.3% loss and 50% recovery) to account for changing credit standards.

Asset Allocation Interactive (AAI) Representation

The following table shows how the methodology is depicted in both the yield + growth and valuation-dependent pricing model breakdowns on our AAI website:

Nominal Government Bonds in Local Currency

		Nominal	Real
Current Yield	Treasury Yield Roll Return	$Current\ Treasury\ Yield + Current\ Roll\ Return$	
Growth	Inflation Credit Losses	$-Expected\ Credit\ Losses$	$-Inflation\ Forecast$ $-Expected\ Credit\ Losses$
Valuation	Change in Treasury Yield Change in Roll Return Treasury Valuation	$Expected\ Average\ Treasury\ Yield - Current\ Treasury\ Yield$ $Expected\ Average\ Roll\ Return - Current\ Roll\ Return$ $Expected\ Treasury\ Valuation\ Return$	

Inflation-Linked Government Bonds

		Nominal	Real
Current Yield	TIPS Yield Roll Return	$Current\ TIPS\ Yield + Current\ Roll\ Return$	
Growth	Inflation	$Inflation\ Forecast$	N/A
Valuation	Change in TIPS Yield Change in Roll Return TIPS Valuation	$Expected\ Average\ TIPS\ Yield - Current\ TIPS\ Yield$ $Expected\ Average\ Roll\ Return - Current\ Roll\ Return$ $Expected\ TIPS\ Valuation\ Return$	

Credit Bonds (Including Hard-Currency Emerging Markets Bonds)

		Nominal	Real
Current Yield	Credit Bond Yield Roll Return	$Current\ Credit\ Bond\ Yield$ $+ Current\ Roll\ Return$	
Growth	Inflation Credit Losses	$-Expected\ Credit\ Losses$	$-Inflation\ Forecast$ $-Expected\ Credit\ Losses$
Valuation	Change in Treasury Yield Change in Treasury Roll Return Treasury Bond Valuation Change in Spread Change in Spread Roll Return Spread Valuation	$Expected\ Average\ Treasury\ Bond\ Yield\ Current\ Treasury\ Yield$ $Expected\ Average\ Treasury\ Roll\ Return\ Current\ Treasury\ Roll\ Return$ $Expected\ Treasury\ Valuation\ Return$ $Expected\ Average\ Yield\ Spread - Current\ Yield\ Spread$ $Expected\ Average\ Spread\ Roll\ Return - Current\ Spread\ Roll\ Return$ $Expected\ Spread\ Valuation\ Return$	

Equity Indices

A capitalization-weighted equity index (e.g., the S&P 500 Index) measures the performance of a basket of stocks weighted by the market capitalization of each company in the index. Within the context of this work we consider the following countries:[14]

1) Developed Markets: Australia, Canada, France, Germany, Hong Kong, Italy, Japan, Netherlands, Spain, Sweden, Switzerland, United Kingdom, United States
2) Emerging Markets: Brazil, China, India, Indonesia, Malaysia, Mexico, Poland, South Africa, South Korea, Taiwan, Thailand, Turkey
3) Style: Growth, Size[15] and Value

Modeling Framework

Consistent with our earlier introduction to the Gordon Growth Model, we decompose the 10-year expected real return earned from investing in a capitalization-weighted equity index, from any country, as follows:

$$Expected\ Equity\ Returns = Dividend\ Yield + Real\ EPS\ Growth\ Forecast + Valuation\ Forecast$$

Although the model calls for the dividend yield at the end of the next investment period, we employ the very high autocorrelation in dividends and compute this component as the ratio between trailing 12-month dividend per share and the current stock market price.

The component $Real\ EPS\ Growth\ Forecast$ captures expected growth in earnings per share, net of inflation. Because our focus is on 10-year returns, we center our estimations on historical trends.

First, for each index we compute the historical trend growth rate of the natural log (ln) of real earnings per share (EPS) by employing a rolling 50-year data window (for indices with less than 50 years of data, we require a minimum of 10 years of observations before computing the first estimate). The natural log is used to remove the natural exponential trend that exists with earnings per share, thus allowing for a more linear fit. We then estimate the historical growth rate by fitting a time trend and label it $\Delta\overline{EPS}_{i,t}$ and using monthly data, specifically,

$$\Delta\overline{EPS}_{i,t} = \left(1 + Slope(\ln(Real\ EPS)\,, Time)\right)^{12} - 1$$

where the function $Slope(\ln(Real\ EPS)\,, Time)$ yields the beta coefficient of regressing the level of the natural log of real EPS onto a time trend.

Second, we average the individual country's forecast with the average growth rate of its group (developed or emerging markets). This approach brings consistency across the various asset classes. For instance, if country i belongs to the emerging markets group, then,

[14] We include these countries, but the model is extensible to other markets as well.
[15] Small cap indices provide additional challenges due to data limitations across countries. See section at the end of the equities section for additional details on small cap modeling.

$$\Delta EPS^*_{j,t} = \frac{1}{2} \times \ \Delta\overline{EPS}_{j,t} + \frac{1}{2} \times \ \Delta\overline{EPS}_{EM,t}$$

Third, we introduce a global upper bound based on our own real GDP projections. If the GDP-weighted average of all countries' expected real EPS growth is greater than world real GDP growth, we scale back proportionally each individual country's expected EPS growth (until the two totals equate). Although EPS growth can be higher than GDP growth over multi-year periods, we do not believe that allowing EPS growth to be greater than GDP growth for a decade is warranted.

The $Valuation\ Forecast$ component is based on mean reversion in the cyclically adjusted earnings yield (CAEY) and follows three steps. The first step consists of computing the long-term ("fair") CAEY at a country level ($\overline{CAEY}_{j,t}$). To do this, we employ an exponentially weighted moving average with a window of 50 years and a half-life of 20 years. We set a minimum data window of 10 years for developed economies and 5 years for emerging markets.

The second step consists of generating country-specific CAEY forecasts by averaging the individual historical level with the group (developed or emerging markets) historical level. For instance, if country j belongs to the emerging markets group, then,

$$CAEY^*_{j,t} = \frac{1}{2} \times \ \overline{CAEY}_{j,t} + \frac{1}{2} \times \ \overline{CAEY}_{EM,t}$$

where a group's CAEY value is computed as an equally weighted average of the countries in the group.

In the third and last step, we assume the current $CAEY_{j,t}$ fully mean reverts to $CAEY^*_{j,t}$ in 20 years. Therefore, the average annual change in valuation is

$$CAEY\ Reversion = \left(\frac{CAEY^*_{j,t}}{CAEY_{j,t}}\right)^{\frac{1}{20}} - 1$$

This horizon choice for the speed of mean reversion approximates the historical experience across developed markets.

Small Cap Modeling Adjustments

Modeling small cap equities has additional challenges due to lack of data across countries, with small cap data in many markets only available in the last decade. To account for this, we focus exclusively on the Russell 2000 and MSCI EAFE Small indices.

The methodology employed for small caps is the same as for large cap assets with the exception of model growth in real earnings per share. Due to the trend in the historical data, we utilize a linear trend instead of a logarithmic trend, which is necessary to account for many periods of negative earnings for small cap indices. Because we are starting with a linear trendline, an additional step is necessary to measure the growth rate of the regression based trendline by looking at the difference between the start and end points relative to the amount of time that has passed.

In addition, since we don't have multiple countries to utilize for cross-sectional averaging, we instead average measure growth in the Russell 2000 with a Bayesian prior of 2.5% growth. We continue to include this prior until the year 2020, at which point the MSCI EAFE Small asset has 20 years of data from which to create a stable estimate of real earnings per share growth for that index. From 2020 onward, the prior is replaced with the estimate of real EPS growth for MSCI EAFE Small. The resulatant real EPS growth estimate is then used for both Russell 2000 and MSCI EAFE Small as we don't have sufficient information to differentiate between growth rates of global small caps.

Small cap value and growth assets provide additional challenges of global data availability. To address this challenge, and maintain our focus on consistency, we make an assumption that value and growth differentials related to the core index are the same regardless of size. We therefore estimate differentials in real earnings per share growth an valuations between large cap value (growth) and core and assume those same differentials apply to small caps[16].

Asset Allocation Interactive (AAI) Representation

The following table shows how the methodology is depicted in both the yield + growth and valuation-dependent pricing model breakdowns on our AAI website:

Equity Indices

		Nominal	Real
Current Yield	Dividend Yield	$Current\ Dividend\ Yield$	
Growth	Real EPS Growth Inflation	$Real\ EPS\ Growth\ Forecast$ $+Inflation\ Forecast$	$Real\ EPS\ Growth\ Forecast$
Valuation	Asset Valuation	$CAEY\ Reversion$	

[16] We acknowledge there can be issues with this broad assumption, but we feel other methods have their own issues and may also introduce biases relative to our focus on internal consistency across models.

Master Limited Partnerships

In the United States, a master limited partnership (MLP) is an entity taxed as a partnership but whose shares are publicly traded on an exchange. Specifically, the shares of an MLP offer an ownership stake in the partnership's business. Therefore, MLPs combine the liquidity of publicly traded securities with the tax benefits of a partnership. MLPs were created in the 1980s and typically operate in the natural resources sector, such as oil and gas, and invest in related infrastructures (primarily pipelines).

MLPs have specific tax implications that arise from their payment of dividends to investors. We ignore taxation in estimating expected returns across all asset classes, and thus we do not consider the tax treatment of MLPs in our estimates for them. For investors utilizing taxable vehicles, the tax rules for MLPs and all assets should be taken into account.

Modeling Framework

In our framework, MLPs are modeled very similarly to other equity indices,

$$Expected\ MLPs\ Returns = Dividend\ Yield + Earnings\ Growth\ Forecast + Valuation\ Forecast$$

We model the three components as follows:

1. Current dividend yield follows the same methodology as for equity indices.
2. Growth in earnings per share (EPS) is estimated by employing an expanding window (with estimates regularly refreshed). Again, we follow the methodology outlined for equity indices (a 50-year window of data, a half-life of 20 years, and a requirement of a minimum of 10 years to estimate any coefficient).
3. The valuation component, however, follows a different methodology from the equity indices, using the dividend yield instead of the CAEY. The choice of dividend yield is driven directly from the implementation challenge of the limited data history available for MLPs. Because of these data limitations, it is not possible to create a meaningful CAEY time series, which requires 10 years of history to create each observation. We are comfortable with this substitution because we are using a fair value estimate, not in absolute terms, but in comparison to the current level.

Although the metric is different, the computation of the fair value and the estimated speed of reversal follows the exact equity methodology. The fair value is computed with an EWMA of the dividend yield (50-year window, half-life of 20 years, and a minimum requirement of 10 years of data), and we assume full mean reversion in 20 years.

Asset Allocation Interactive (AAI) Representation

The following table shows how the methodology is depicted in both the yield + growth and valuation-dependent pricing model breakdowns on our AAI website:

MLPs

		Nominal	Real

Current Yield	Dividend Yield	$Current\ Dividend\ Yield$	
Growth	Real EPS Growth Inflation	$Real\ EPS\ Growth\ Forecast$ $+Inflation\ Forecast$	$Real\ EPS\ Growth\ Forecast$
Valuation	Asset Valuation	$Dividend\ Yield\ Reversion$	

Real Estate Investment Trusts

A real estate investment trust (REIT) is a company that owns and may operate real estate investments. A REIT can invest in different types of commercial real estate, including, but not limited to, apartments, data centers, gaming establishments, hotels, hospitals, offices, shopping centers, and warehouses.

From a portfolio perspective, a REIT is an income-producing asset that must pay out its earnings as dividends. REITs tend to be sensitive to interest rates because they acquire, own, and operate properties and use debt financing within their real estate portfolios. Interest rate expectations affect the valuation of a REIT.

Modeling Framework

We model REITs as having both equity and bond characteristics. The model is an application of the equity framework,

$$Expected\ REITS\ Returns = Dividend\ Yield + DPS\ Growth\ Forecast + Valuation\ Forecast$$

where DPS stands for real dividend per share. We modeling the three components as follows:

1. Dividend yield is modeled using the most recent trailing 12-month dividend yield, consistent with the equity model.
2. Growth in dividend per share is estimated by employing an expanding window (with estimates regularly refreshed). We follow the methodology outlined for equity indices (50-year window, 20-year half-life, and a minimum requirement of 10 years). Because REITs must pay out their earnings as dividends, we utilize the dividend data. In addition, we start the measurement window in 1992, the year the laws governing REITs changed, and REIT ownership within portfolios began to grow.
3. The valuation component is characterized as follows:

$$REITS\ Spread = DY\ - (10Y\ Real\ Yield + \frac{1}{2}[BBB\ Spread] + \frac{1}{2}[S\&P\ CAEY\ Spread])$$[17]

We employ an EWMA to estimate the fair value of a yield spread (50-year window, half-life of 20 years, and a minimum of 10 years of data),

$$Expected\ REITS\ Spread_{Long\ Run} = EWMA(Past\ REITS\ Spreads)$$

and use monthly regressions to estimate the speed of convergence,

$$\begin{aligned} &Monthly\ Change\ in\ REITS\ Spread \\ &\qquad = \hat{\rho} \times \left(REITS\ Spread - Expected\ REITS\ Spread_{Long\ Run}\right) \end{aligned}$$

The term $\hat{\rho}$ measures the speed of mean reversion over the subsequent month; that is, by how much today's difference between the $Spread$ and its long-term fair value is "closed" by next month's change in the $Spread$. Lastly, we impose our priors and bound this value to be within -0.015 and -1.0.

[17] The BBB Spread is computed as the difference between BBB bond yields and maturity-matched Treasury yields, whereas the S&P CAEY Spread is the difference between the cyclically adjusted earning yield for the S&P500 and the 10-year Treasury real yield.

Asset Allocation Interactive (AAI) Representation

The following table shows how the methodology is depicted in both the yield + growth and valuation-dependent pricing model breakdowns on our AAI website:

REITS

		Nominal	Real
Current Yield	Dividend Yield	$Current\ Dividend\ Yield$	
Growth	Real DPS Growth Inflation	$Real\ DPS\ Growth\ Forecast$ $+ Inflation\ Forecast$	$Real\ DPS\ Growth\ Forecast$
Valuation	Asset Valuation	$REITS\ Spread\ Reversion$	

Commodity Futures and Indices

A commodity futures contract is an agreement to buy or sell a particular commodity at a future date and predetermined price, and it provides investors with access to these markets without having to trade spot commodities (e.g., barrels of oil, tons of metals, and so forth). Most investors have no need or interest in taking delivery (or delivering) of the underlying commodity, therefore, commodity contracts are usually liquidated before the delivery date. Because we are focused exclusively on modeling derivatives contracts that do not require full investment until delivery, in order to estimate total return in the future, we need to define the collateral used to finance the positions.

The modeling methodology builds on two drivers of return. First, a commodity futures investor earns a return on the collateral plus the carry, which can be understood as the yield component of the return (see Koijen et al. [2018] for a detailed overview of the significance of carry). In addition, the investor may earn or lose from the appreciation or depreciation, respectively, in the spot price of the underlying commodity asset. As for foreign currencies, we model long-term price movements by employing reversal (value) indicators.

Modeling Framework

We decompose the 10-year expected returns earned from investing in a commodity future contract as follows:

$$Expected\ Commodity\ Futures\ Return = Collateral\ Forecast + Spot\ Forecast + Carry\ Forecast$$

We adopt common convention and assume collateral is invested in short-term bills; see the section on Treasury bills for details on how we forecast these returns.

To predict the change in valuation of a commodity futures contract, the $Spot\ Forecast$, we proceed using the following five steps:.

1. For each commodity, we compute a real *reversal indicator* by employing an EWMA of past real spot returns (the moving average uses 10 years of data and a half-life of five years).
2. Equipped with the reversal metric, at the end of every month we estimate its predictive power on the subsequent 10-year real spot return,

   $$Future\ 10Y\ Returns = \hat{\beta}_t \times Reversal_t$$

   The resulting estimated predictive coefficient $\hat{\beta}$ can be interpreted as the magnitude of reversal. To estimate this model, we run pooled regressions across the commodity sectors of energy, grains, industrial metals, precious metals, livestock, and softs. At a minimum, we require five years of overlapping data to estimate a parameter.
3. To avoid counterintuitive results driven by outliers, we bound the values of $\hat{\beta}_t$ to be within −0.2 and −1.0. Instead of using the direction of the results from the regressions, which can be noisy and sometimes unintuitive, we introduce priors. Our prior is that at least 20% reversal is a fair estimate for subsequent decades, whereas values greater than 100% reversal seem to be inadequate expectations for a 10-year horizon. We label the resulting coefficient $\hat{\beta}_{t,B}$.
4. In addition, we take a 12-month average of the magnitude of reversal to smooth its profile. We obtain

   $$\beta_t^* = \sum_{\tau=0}^{11} \hat{\beta}_{t-\tau,B}$$

5. Lastly, $Real\ Spot\ Forecast$ is defined as the most recent reversal indicator times the predicted magnitude of reversal,

   $$Real\ Spot\ Forecast = \beta_t^* \times Reversal_t$$

Carry capture is the expected return from rolling over futures to maintain consistent exposure. For a curve that is upward sloping, in contango, the carry return will be negative because the price of the future contract is expected to fall toward the price of the next nearer-to-maturity contract. If the curve is downward sloping, in backwardation, the opposite is true and the carry return is positive.

Carry expectations are computed as the slope of the front-end of the curve. Because most futures contracts mature over only a few months, in order to estimate a 10-year carry return, we need to make assumptions about the future slope of the futures curve. To do so, we employ an EWMA of past carry returns to form our carry expectations and scale this average by a coefficient of 0.3 in order to capture the fact that commodity futures tend to revert “against” carry. The EWMA is calibrated with a data window of 10 years and a half-life of five years.

If a commodity is deemed to be seasonal, we take a 12-month average of the carry signal to “purge” any predictable fluctuations in the slope of the futures curve. We classify the following commodity contracts as seasonal: heating oil, gasoil, natural gas, gasoline, corn, Kansas wheat, soybeans, soybean meal, soybean oil, wheat, feeder cattle, live cattle, lean hogs, and sugar. All other contracts are nonseasonal.

Commodity Indices

Commodity indices are simply weighted baskets of commodity futures contracts, many of which are rebalanced on a periodic (usually annual) basis. In addition to calculating the expected return of an index as the weighted average of thew underlying futures expectations, the long-run expected returns earned for the index must also account for the rebalance premium. Specifically, let the weight of commodity j in index K be given by $w_{j,t}^{J}$, where $\sum_{j=1}^{J} w_{j,t}^{K} = 1$ and J is the total number of commodity contracts in the index. Then, it follows that the returns of the index are given by

$$Expected\ Commodity\ Index\ Returns =$$
$$= \underbrace{\sum_{j=1}^{J} E_t\left[w_{j,t}^{K} R_{j;t\to t+10Y}\right]}_{\text{Weighted Futures Returns}} + \underbrace{\frac{1}{2}\sum_{j=1}^{J} w_{j,t}^{K}\left(\sigma_{j;t-1,t-120}^{2} - \sigma_{K;t-1,t-120}^{2}\right)}_{\text{Rebalance Premium}}$$

where σ_j^2 is the volatility of the individual commodity, whereas σ_K^2 is the volatility of the index.

Asset Allocation Interactive (AAI) Representation

The following table shows how the methodology is depicted in both the yield + growth and valuation-dependent pricing model breakdowns on our AAI website:

Commodities

		Nominal	Real
Current Yield	T-Bill Yield Commodity Carry[18]	$TBill\ Yield$ $Carry\ Forecast$	
Growth	Inflation	N/A	$-Inflation\ Forecast$
Diversification	Asset Diversification	$Rebalance\ Premium$	
Valuation	Change in T-Bill Yield Asset Valuation	$Expected\ TBill\ Yield - Current\ TBill\ Yield$ $Real\ Spot\ Forecast$	

Modeling Alternatives

In addition to modeling public market asset classes, we also model expected returns for private asset classes—leveraged buyout private equity, venture capital, and private commercial real estate—and for trading strategies such as a long/short equity hedge fund. Unlike public market asset classes, which have a wealth of available data, with the exception of commercial real estate, indices of alternatives suffer from data-quality issues, such as survivorship bias, ex post updating, and selection bias.

Therefore, instead of basing our models on existing indices of alternatives, we seek to replicate these asset classes and trading strategies by building our own indices using a bottom-up approach to select individual securities. We then measure the systematic exposures and alpha of the custom indices as the basis for expected returns.

In the case of alternatives, we do not seek to measure the return of any particular manager, or even managers in general, but instead focus on creating expected returns for the asset class or the trading strategy. This means we do not consider fees in our expectations. Additionally, the volatility of alternatives is often understated because we

[18] Unlike other assets for which the yield component is a current yield, because commodity futures are short-life assets relative to our 10-year return expectation and because commodity carry tends to be volatile, we opt for our average carry forecast over the 10 years instead of the carry based on the current term structure.

must rely on periodic appraisal values instead of the daily mark to market available in public markets. We aim to achieve a like-for-like comparison, therefore, we strive to show implied market volatility for these strategies, which is possible through our custom index creation process.

Modeling Commercial Real Estate

In modeling of private commercial real estate expected returns, we utilize data from the National Council of Real Estate Investment Fiduciaries, which publishes quarterly fundamentals on property across the United States. Our analysis is based on the national commercial real estate market categorized by type—apartments, industrial, office and retail—and by region—East, Midwest, South and West.

Modeling Framework

We leverage the research of Ilmanen et al. (2019) and decompose the 10-year expected real return earned from investing in commercial real estate as follows:

$$Expected\ CRE\ Returns = Cash\ Flow\ Yield + Real\ NOI\ Growth\ Forecast + Valuation\ Forecast$$

Cash-flow yield is determined based on the current capitalization rate of properties in the type or region of interest, adjusted for capital expenditures. Because property is a real asset, we consider maintenance costs in determining the yield to the investor.

The $Real\ NOI\ Growth\ Forecast$ captures expected growth in net operating income (NOI) net of inflation. Because we are estimating 10-year returns, we are able to utilize historical trends, which over shorter horizons can massively under- or overestimate returns.

First, for each index we compute the historical trend growth rate of the log of net operating income by employing a rolling 50-year data window (for indices with less than 50 years of data, we implement a lower bound requirement of 10 years). The log is used to remove the natural exponential trend that can exist with NOI.

Lastly, the $Valuation\ Forecast$ component is based on mean reversion in the capitalization rate, net of capex, to our estimate of fair value. Long-term (“fair”) cap rates are based on an exponentially weighted moving average with a window of 50 years and a half-life of 20 years. We assume full mean reversion to fair-value cap rates in 20 years.

Asset Allocation Interactive (AAI) Representation

The following table shows how the methodology is depicted in both the yield + growth and valuation-dependent pricing model breakdowns on our AAI website:

Commercial Real Estate

		Nominal	Real
Current Yield	Cap. Rate (net of Cap. Ex.)	$Cash\ Flow\ Yield$	
Growth	Real NOI Growth Inflation	$Real\ NOI\ Growth\ Forecast + Inflation\ Forecast$	$Real\ NOI\ Growth\ Forecast$
Valuation	Asset Valuation	$Cash\ Flow\ Yield\ Reversion$	

Modeling Leveraged Buyout Private Equity

Private equity (PE) is an alternative type of investment for ownership of firms not listed on a public exchange. PE investment vehicles are funds in which investors can pool their money and take stakes in privately held companies. In the particular case of a leveraged buyout (LBO), a fund buys the shares in a public company and then leads to its delisting from the exchange.

By building on the work of Gompers et al. (2016) and Korteweg (2019), we approximate private equity returns as an investment in the public market with certain factor exposures (betas), such as value and size. Because leveraged buyout PE investments may make use of leverage, we also account for borrowing cost (debt).

$$
\begin{aligned}
&Expected\ LBO\ Returns \\
&\qquad = Expected\ TBill\ Rate + Betas \times Expected\ Factor\ Returns \\
&\qquad - Expected\ Cost\ of\ Debt
\end{aligned}
$$

Strategy Replication

The first step in our process is to create an investment strategy to synthetically replicate the returns to a strategy of leveraged buyouts. Our strategy utilizes public company fundamentals (from Compustat) and returns (from The Center of Research in Security Prices, CRSP), and empirical facts about the fundamentals of LBO targets to create the synthetic series. We utilize those empirical facts to filter the universe of publically available companies to approximate a target list of LBO targets.

Our goal is not to identify companies that were taken private in LBOs, but to identify public companies with similar characteristics such that we can approximate the return of companies that were taken private. An alternative approach is to look at, and model, individual LBO deals and extrapolate for that outcome. We focus, however, on the more-straightforward approach using data that are available to most investors.

We first select the starting universe by filtering the security universe based on seven characteristics. The choice of these features is consistent with the work of Gompers et al. (2016) and Korteweg (2019). The following table lists the seven filters:

Filter	Rule
Industry	Communications, Consumer Cyclicals, Consumer Discretionary, Heathcare, Industrials, Technology
Size	50^{th} percentile (per sector) < Market Cap < 70^{th} percentile (per sector)[19]
Value	Book to Market > Max(0, 50th percentile (per sector))
Operating Profitability (a)	0 < EV/EBITDA < 50th percentile (per sector)
Operating Profitability (b)	FCF/EV > Max(0, 50th percentile (per sector))
Low Investment	Asset Total Change < 50th percentile (per sector)
High Debt Service	Debt Coverage Ratio and Debt Coverage Ratio > Max(0, 50th percentile (per sector))

These filters are applied at every date, yielding a monthly group of public companies that have the risk characteristics of LBO targets.

Second, we translate the filters to portfolio weights. Specifically, we form a cap-weighted portfolio of the selected companies, where each company's individual weight is a function of its own market capitalization relative to the capitalization of the entire group of selected companies.

Third, we take a 60-month average of the portfolio weights, based on an assumption of a five-year holding period of the target company.

[19] The CRSP/Compustat data had an inherent small-cap bias, with a high concentration of companies with a market cap less than $500 million. To account for this, we raised the percentile. Because company characteristics vary across industries, all of these factors were filtered on a per sector basis, meaning that the percentile was calculated relative to companies within the same sector.

Fourth, the LBO assumes large amounts of debt to enhance the gross return (e.g., see Ilmanen et al. [2019]). Hence, we need to account for 1) the amount of leverage and 2) the cost of leverage. Specifically, we employ the Modigliani–Miller theorem (1958),

$$r_{Levered} = r_{Unlevered} + \frac{D}{E} \times (r_{Unlevered} - k_d)$$ [20]

to ascertain the levered return.

We roughly approximate the cost of debt k_d with the yield of the Credit Suisse Leveraged Loan Index from January 31, 1991, to December 31, 2006, and the yield of the JP Morgan Leveraged Loan Index (LILI Index) from January 31, 2007, onward. Historically, the debt-to-equity ratio of LBOs has been time varying, with a gradual decline from about 3 in the 1990s to about 1 today, which is the number we will employ going forward (more details can be found in Brown [2021]).

Modeling Framework

Equipped with the PE portfolio generated by steps one to four, we estimate a factor model meant to capture the risk exposures, or investment styles, that characterize PE. Specifically, we run 10-year rolling regressions of the gross levered cap-weighted portfolio return, $\left(1 + \frac{D}{E}\right) \times R_U$, on the realized returns of the equity factors market, size, value, profitability, and investment plus momentum. The rolling regressions provide us with continuously updated betas (where β_i indicates the exposure to factor i). These exposures are combined with factor expected returns – see description at the end of the Alternative sections for details on equity factor expected return modeling.

[20] In the Modigliani–Miller theorem, r_L is the levered return, while r_U *and* k_d are the unlevered return and cost of debt, respectively.

Asset Allocation Interactive (AAI) Representation

The following table shows how the methodology is depicted in both the yield + growth and valuation-dependent pricing model breakdowns on our AAI website:

LBO

		Nominal	Real
Current Yield	Yield Cost of Debt	$Current\ Yield + \sum\beta_i \times E[F]_{yield,i}$ $-Real\ Bank\ Loan\ Yield$	
Growth	Real Growth Inflation	$\sum\beta_i \times E[F]_{growth,i}$ $+Inflation\ Forecast$	$\sum\beta_i \times E[F]_{growth,i}$
Valuation	Change in T-Bill Yield Asset Valuation	$Expected\ TBill\ Yield - Current\ TBill\ Yield$ $\sum\beta_i \times E[F]_{ivaluation,}$	

Modeling Venture Capital

Venture capital (VC) is a type of private equity investment that focuses on financing startup companies and young, small businesses that have long-term growth potential. Therefore, our modeling approach mirrors the one outlined for LBO private equity investments, except for two key differences: 1) a different choice of characteristics to filter the universe of firms and 2) no leverage assumptions.[21]

Strategy Replication

With the exception of two differences, our methodology follows the same steps detailed for LBO private equity. The first difference pertains to the choice of filtering characteristics. The characteristics we use are motivated by the research of Korteweg and Sorensen (2010), Korteweg and Nagel (2016), and Ilmanen et al. (2019).[22] Specifically, we use the following filters:

[21] Venture capital funds have highly skewed return distributions, making them difficult to model. Whereas venture capital funds and LBOs exploit different market inefficiencies, they suffer from the same data modeling complications. The underlying thesis behind all of our CMEs emphasizes the importance of precision over accuracy, as the relative performance of asset classes is, by definition, the source of all excess return. Therefore, in the spirit of precision, we model venture capital funds using the same factor-based approach that we use to model LBOs.

[22] As noted by Ilmanen et al. (2019): “Venture capital targets are more likely to be growth companies.”

Filter	Rule
Industry	Communications, Consumer Cyclicals, Healthcare, Technology[23]
Size	20^{th} percentile (per sector) < Market Cap < 40^{th} percentile (per sector)
Growth (a)	0 < B/M < 50^{th} percentile (per sector)
Growth (b)	Revenue Growth > Max(0, 50th percentile (per sector))
High Investment	Asset Total Change > 50th percentile (per sector)

These filters are applied at every date, yielding a monthly group of public companies that have the features of VC's targets.

Asset Allocation Interactive (AAI) Representation

The following table shows how the methodology is depicted in both the yield + growth and valuation dependent pricing model breakdowns on our AAI website:

Venture Capital

		Nominal	Real
Current Yield	Yield	$Current\ Yield + \sum\beta_i \times E[F]_{yield,i}$	
Growth	Real Growth Inflation	$\sum\beta_i \times E[F]_{growth,i}$ $+Inflation\ Forecast$	$\sum\beta_i \times E[F]_{growth,i}$
Valuation	Change in T-Bill Yield Asset Valuation	$Expected\ TBill\ Yield - Current\ TBill\ Yield$ $\sum\beta_i \times E[F]_{growth,i}$	

Modeling Long/Short Equity Hedge Funds

Long/short equity is the largest class of hedge funds and seeks to take both long and short positions in equity securities while maintaining a net positive long exposure. Although the short portfolio can contain individual securities, public indices, such as the S&P 500 and Russell 2000 in the United States, are used to manage the equity beta of the portfolio.

To add validity to our custom hedge fund index creation, we studied holdings of a cross-section of long/short hedge funds to identify average positioning, as well as dynamic

[23] The Venture Capital Deals Dashboard, using data from Dow Jones VentureSource, made available by the *Wall Street Journal*, we observe that VC deals seem to be dispersed across fewer sectors than LBOs. Chart available at https://graphics.wsj.com/venture-capital-deals/

positioning, with respect to industry sectors, factor exposures, and gross exposure. We then created a custom security selection algorithm in line with those metrics.

Custom Index Design Overview

The custom index design utilizes the CRSP/Compustat universe of US stocks for the long portfolio, and the S&P 500 and Russell 2000 indices for the short universe.

We create the long portfolio by first scoring all securities across four metrics: industry sector, size (market cap), value (book-to-price ratio), and asset effectiveness (return on assets). The scoring works by classifying each security into one of 10 sectors, three size buckets, two value buckets, and two asset buckets, creating a possible total of 120 unique groupings of stocks (clusters) in every time period.

In order to create a diversified portfolio, one security is selected from every cluster in each time period, meaning the portfolio could have a maximum of 120 holdings. Consistent with our study of long/short managers, the selected security is the security in each bucket with the highest trailing 12-month momentum. To limit turnover, hurdle rates are defined to keep existing securities in the portfolio as well as simply not turning over the portfolio every month. Once securities are selected, they are weighted based on desired sector weights determined by our cross-sectional analysis of actual long/short holdings.

The short portfolio contains two positions in two indices and is meant to control the exposure and the equity beta of the portfolio based on market momentum, with an average net exposure close to 35%.

Modeling Framework

The expected return of the long/short hedge fund strategies is determined through a regression analysis of our custom index against a set of standard equity factors. Specifically, we run 10-year rolling regressions of the long/short hedge fund strategy return on the realized returns of the equity factors market, size, value, profitability, and investment plus momentum. The rolling regressions provide us with continuously updated betas (where β_i indicates the exposure to factor i). These exposures are combined with factor expected returns – see description at the end of the Alternative sections for details on equity factor expected return modeling.

Unlike leverage buyouts and venture capital, the long/short hedge fund also displays a statistically significant alpha representing the trading alpha from this trading strategy. Rather than using the regression alpha directly, as it is impact by the historical estimation window, we shrink the trading alpha in half as par tof the go forward expectation.

Asset Allocation Interactive (AAI) Representation

The following table shows how the methodology is depicted in both the yield + growth and valuation dependent pricing model breakdowns on our AAI website:

Venture Capital

<table>
<tr><th></th><th></th><th>Nominal</th><th>Real</th></tr>
<tr><td>Current Yield</td><td>Yield</td><td colspan="2">$Current\ Yield + \sum\beta_i \times E[F]_{yield,i}$</td></tr>
<tr><td>Growth</td><td>Real Growth
Inflation</td><td>$\sum\beta_i \times E[F]_{growth,i}$
$+Inflation\ Forecast$</td><td>$\sum\beta_i \times E[F]_{growth,i}$</td></tr>
<tr><td>Valuation</td><td>Change in T-Bill Yield
Asset Valuation</td><td colspan="2">$Expected\ TBill\ Yield - Current\ TBill\ Yield$
$\sum\beta_i \times E[F]_i$</td></tr>
<tr><td>Alpha</td><td>Trading Alpha</td><td colspan="2">$\frac{Historical\ Trading\ Alpha}{2}$</td></tr>
</table>

Modeling Long/Short Equity Factors for Alternatives

In cases where we have modeled the long and short legs of a particular equity factor, the generation of the expected return is very straightforward. This is the case with both HML (value) and SMB (size) where we simply take the difference of the Value and Growth (or Large and Small) asset classes to generate the factor expected return. Because we also have return building blocks, for these assets, the building block returns of the equity factors are also trivial to generate. [24]

For factors were we currently don't model the relevant long and short legs of the factor, nor the factor directly, for example investment, momentum, and profitability factors, we used a scaled version of historical returns of the factors. For the currently modeled alternatives, the exposure to these factors tends to be smaller than the exposure to the market, value and size factors, such that the impact of this simple historical average method is moderate.

[24] We also note that given the disaggregation of factor expected returns into building blocks is not exact, at times there can be unintuitive decompositions (negative yield for example); however, the important point is that the total is as intended even if the disaggregation can be noisy at times.

Additionally, with respect to factor expected-return disaggregation into our yield, growth, and valuation building blocks, we approximate that on average 25% of factor expected returns are from yield, 25% from growth, and 50% from valuation. Actual values vary by factor and were measured empirically, but these averages provide directional intuition.

Modeling Currency Translations

When investing abroad, an investor must decide how to manage their foreign exchange (FX) exposures: they can let them be affected by foreign currency fluctuations or they can choose to hedge them via forward markets. These two options apply to *any* of the asset classes included in this methodology document, and they entail different costs and risks.

An investor who does not hedge possible fluctuations in bilateral exchange rates faces *FX risk*. That is, when the investor repatriates their capital after investing in foreign assets, their home currency may have appreciated or depreciated against the foreign currency. In the case of a home-country currency appreciation, the investor would face negative FX returns from their foreign investments. We generalize the expected return formulation as

$$Expected\ Unhedged\ Asset\ Return = Expected\ Asset\ Return + Expected\ FX\ Return$$

where $Expected\ FX\ Return$ is positive only if the foreign currency involved in the investment appreciate against the home currency. *Expected Asset Return* represents the local-currency expected return for any asset class described in this document.

An investor who chooses to hedge their foreign currency exposures should not incur any significant FX risk, but may incur hedging costs.[25] These hedging costs are a function of the risk-free rate differential between the home and foreign countries. The intuition is simple. Hedging FX risk involves setting today the future price of a bilateral exchange rate. Hence, the actors involved in the transaction will aim to eliminate any pure arbitrage opportunity, such as harvesting yield differentials between risk-free (T-bill) investments without incurring any FX risk. This condition, known as *covered interest rate parity*, implies that the returns from exchanging a currency today and simultaneously buying an offsetting position via a forward contract will result in the risk-free differential between the home and foreign rates. Hence, we generalize the hedged expected returns as follows:

$$Expected\ Hedged\ Asset\ Returns = Expected\ Asset\ Return + Expected\ Hedging\ Costs$$

Modeling Framework: Unhedged Foreign Investments

Consider an investor who makes a generic foreign investment without hedging their FX exposures. We define the one-period unhedged return R_{t+1}^{U} as

[25] FX hedging can minimize an investor's exposure to FX, but might not completely eliminate it, because the size of the capital resulting from the foreign investment is not known with certainty as a result of investment risk. Hence, in order for an investor to reduce the mismatch between the size of the capital invested and the FX exposure, they would need to continuously hedge their evolving pool of foreign investments.

$$R_{t+1}^{U} = (1 + R_{t+1})\frac{S_{t+1}}{S_t} - 1$$

where R_{t+1} represents the local-currency return earned from the asset, and define S_t as the bilateral exchange rate or *spot rate* at time t, which indicates how many units of the home currency can be purchased with one unit of the foreign currency. From this equation, we can see that the investor earns a positive FX return if the home currency depreciates (e.g., S moves from 1.20 to 1.25) and earns a negative return if it appreciates. By rearranging the return equation and taking logs, we can approximate it as the simple sum of two components,

$$R_{t+1}^{U} \cong R_{t+1} + \Delta S_{t+1}$$

where $\Delta S_{t+1} = \frac{S_{t+1} - S_t}{S_t}$ is the percentage change in the bilateral exchange rate. This expression can be generalized to any investment horizon by simply changing the subscripts in the equation. Therefore, the unhedged expected returns equal the sum of the local asset expected returns and the expected FX returns.

The expectations for the local asset returns are outlined in the relevant asset-class section. Hence, we need to determine the expected FX returns. Our forecasting methodology builds on two economic foundations. First, we apply the *relative purchasing power parity* (RPPP) theory, which predicts that the inflation rate differentials between two countries should drive their bilateral exchange rate. As discussed by Taylor and Taylor (2004), the RPPP tends to hold in the long run, which is our horizon of interest. Second, we note that real exchange rates tend to exhibit reversal, as Asness, Moskowitz, and Pedersen (2013) documented. We can conveniently decompose these two channels from our original FX returns expression as

$$Nominal\ FX\ Returns = \frac{\Delta S_{t+1}}{S_t} = \underbrace{\frac{\Delta RS_{t+1}}{RS_t}}_{Real\ FX\ Returns} + \underbrace{\pi_{\underset{t+1}{Home}} - \pi_{\underset{t+1}{Foreign}}}_{FX\ Inflation\ Differential}$$

By definition, *real FX returns*, $\frac{\Delta RS_{t+1}}{RS_t}$, equals *nominal FX returns* minus the *inflation differential*, where $\pi_{\underset{t+1}{Home}}$ indicates the one-period inflation rate in the investor's home country and $\pi_{\underset{t+1}{Foreign}}$ is the inflation rate in the local country of the asset.

The section on inflation discusses the modeling assumptions that underpin our long-term inflation expectations. To model $Real\ FX\ Returns$, we proceed using the following five steps that mirror those already outlined for commodity futures:

1. For each country, we compute a real *reversal indicator* by employing an EWMA of past real spot returns (the moving average uses 10 years of data and a half-life of five years).
2. Equipped with the reversal metric, at the end of every month we estimate its predictive power on the subsequent 10-year real spot return:

$$Future\ 10Y\ Returns = \hat{\beta}_t \times Reversal_t$$

The resulting estimated predictive coefficient $\hat{\beta}$ can be interpreted as the magnitude of reversal. To estimate this model, we run separate pooled regressions for developed and emerging markets. Developed markets tend to be "better behaved" from a data perspective, and we prefer those results not be impacted by emerging market data, which are more volatile.

3. To avoid counterintuitive results driven by outliers, we bound the values of $\hat{\beta}_t$ to be within −0.2 and −1.0. Instead of using the direction of the results from the regressions, which can be noisy and sometimes unintuitive, we introduce priors. Our prior is that at least 20% reversal is a fair estimate for subsequent decades, whereas values greater than 100% reversal seem to be inadequate expectations for a 10-year horizon. We label the resulting coefficient $\hat{\beta}_{t,B}$.
4. In addition, we take a 12-month average of the magnitude of reversal to smooth its profile. We obtain

$$\beta_t^* = \sum_{\tau=0}^{11} \hat{\beta}_{t-\tau,B}$$

5. Lastly, $Real\ Spot\ Reversion$ is defined as the most recent reversal indicator times the predicted magnitude of reversal,

$$Real\ Spot\ Reversion = \ \beta_t^* \times Reversal_t$$

Asset Allocation Interactive (AAI) Representation

The following table shows how the methodology is depicted in both the yield + growth and valuation dependent pricing model breakdowns on our AAI website:

Currency Translation

		Nominal	Real
Current Yield	Yield	N/A	
Growth	FX Inflation Differential	$Inflation\ Forecast_{Home} - Inflation\ Forecast_{Foreign}$	N/A
Valuation	Real FX Valuation	$Real\ Spot\ Reversion$	

Note: *Home* refers to the home country of the investor, and *Foreign* refers to the country in which the investment is made. The AAI tables presented in earlier sections of this document detail the combination of return components needed to construct a *local* expected return in real or nominal terms. For those tables, the "Nominal" and "Real" column headers represent the desired expected return to construct. In this section, the column headers refer to the "Nominal" or "Real" state of the expected returns to be currency converted, and the associated "Nominal" or "Real" state of the currency converted result.

Hedging Cost Translation

An investor can choose to hedge their foreign currency exposure any time they invest in an asset denominated in a currency other than their home currency. The decision to hedge is especially prevalent when the volatility arising from the currency exposure is higher than the volatility of the asset's price (e.g., global developed bonds).

As explained earlier, an investor may face a cost when hedging their FX exposures by investing in forward contracts. To appreciate this point, define F_t as the price of a forward contract at time t that will expire after one period at time $t+1$. The forward contract gives the holder the right to exchange a currency at a pre-determined rate and at a pre-determined time. By following Koijen et al. (2018), the no-arbitrage price equation of a currency forward contract is

$$F_t = S_t \frac{(1+i_t^{Home})}{(1+i_t^{Foreign})}$$

where i_t^{Home} indicates the investor's home-country risk-free rate observed at time t, and $i_t^{Foreign}$ is the corresponding risk-free rate in the country of the asset. We define the one-period hedged return R_{t+1}^H from investing in a foreign asset with return R_{t+1}

$$R_{t+1}^H = (1+R_{t+1})\frac{F_t}{S_t} - 1$$

The key difference with respect to unhedged returns is the introduction of the forward price, F_t, to the equation. Next, we can substitute the expression of the forward price and take the log of the equation to obtain

$$R_{t+1}^H \cong R_{t+1} + \underbrace{i_t^{Home} - i_t^{Foreign}}_{Hedging\ Costs}$$

$$= R_{t+1} + \underbrace{r_t^{Home} - r_t^{Foreign}}_{Real\ Hedging\ Costs} + \underbrace{\pi_{t+1}^{Home} - \pi_{t+1}^{Foreign}}_{Hedging\ Inflation\ Differential}$$

hence, hedged returns are the sum of the foreign asset's local-currency returns plus the costs of hedging the FX exposure. Above, we use the notation $r_t^{Home} = i_t^{Home} - \pi_{t+1}^{Home}$ to indicate the investor's home country real risk-free rate, and $r_t^{Foreign} = i_t^{Foreign} - \pi_{t+1}^{Foreign}$ to indicate real risk-free rate in the country of the asset.

Asset Allocation Interactive (AAI) Representation

The following table shows how the methodology is depicted in both the yield + growth and valuation dependent pricing model breakdowns on our AAI website:

Hedging Costs

		Nominal	Real

Current Yield	Real Hedging Cost	$(Current\ TBill\ Yield_{Home\ TBill} - InflationForecast_{Home}) - (Current\ TBill\ Yield_{Foreign\ TBill} - InflationForecast_{Home})$	
Growth	Hedging Inflation Differential	$Inflation\ Forecast_{Home} - Inflation\ Forecast_{Foreign}$	N/A
Valuation	Change in Real Hedging Costs	$(Expected\ TBill\ Yield_{Home} - Current\ TBill\ Yield_{Home}) - (Expected\ TBill\ Yield_{Foreign} - Current\ TBill\ Yield_{Foreign})$	

Note: *Home* refers to the home country of the investor, and *Foreign* refers to the country in which the investment is made. The valuation component does not include inflation because the expected and current inflation rates in each country are modeled to be the same. The AAI tables presented in earlier sections of this document detail the combination of return components needed to construct a *local* expected return in real or nominal terms. For those tables, the “Nominal” and “Real” column headers represent the desired expected return to construct. In this section, the column headers refer to the “Nominal” or “Real” state of the expected returns to be currency converted and the associated “Nominal” or “Real” state of the currency converted result.

References and Additional Reading

For interested readers, we offer a selection of highly important and influential papers that have aided us in our model design.

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The asset classes are represented by broad-based indices which have been selected because they are well known and are easily recognizable by investors. Indices have limitations because indices have volatility and other material characteristics that may differ from an actual portfolio. For example, investments made for a portfolio may differ significantly in terms of security holdings, industry weightings and asset allocation from those of the index. Accordingly, investment results and volatility of a portfolio may differ from those of any index referenced. The indices noted in this in this interactive site are unmanaged, are not available for direct investment, and are not subject to management fees, transaction costs or other types of expenses that a portfolio may incur. In addition, the performance of the indices reflects reinvestment of dividends and, where applicable, capital gain distributions. Therefore, investors should carefully consider these limitations and differences when evaluating the index performance.

No investment process is risk free and there is no guarantee of profitability; investors may lose some or all of their investments. No investment strategy or risk management technique can guarantee returns or eliminate risk in any market environment. Diversification does not guarantee a profit or protect against loss. Investing in foreign securities presents certain risks not associated with domestic investments, such as currency fluctuation, political and economic instability, and different accounting standards. This may result in greater share price volatility. The prices of small- and mid-cap company stocks are generally more volatile than large-company stocks. They often involve higher risks because smaller companies may lack the management expertise, financial resources, product diversification and competitive strengths to endure adverse economic conditions.

Bond prices fluctuate inversely to changes in interest rates. Therefore, a general rise in interest rates can result in the decline of the value of your investment. High-yield bonds, also known as junk bonds, are subject to greater risk of loss of principal and interest, including default risk, than higher-rated bonds. Investing in fixed-income securities involves certain risks such as market risk if sold prior to maturity and credit risk especially if investing in high-yield bonds which have lower ratings and are subject to greater volatility. All fixed-income investments may be worth less than original cost upon redemption or maturity. Income from municipal securities is generally free from federal taxes and state taxes for residents of the issuing state. While the interest income is tax-free, capital gains, if any, will be subject to taxes. Income for some investors may be subject to the federal alternative minimum tax (AMT).

There are special risks associated with an investment in real estate, including credit risk, interest-rate fluctuations and the impact of varied economic conditions. Distributions from REIT investments are taxed at the owner's tax bracket.

Hedge funds or alternative investments are complex, speculative investment vehicles and are not suitable for all investors. They are generally open to qualified investors only and carry high costs and substantial risks and may be highly volatile. There is often limited (or even nonexistent) liquidity and a lack of transparency regarding the underlying assets. They do not represent a complete investment program. The investment returns may fluctuate and are subject to market volatility so that an investor's shares, when redeemed or sold, may be worth more or less than their original cost. Hedge funds are not required to provide investors with periodic pricing or valuation and are not subject to the same regulatory requirements as mutual funds. Investing in hedge funds may also involve tax consequences. Speak to your tax advisor before investing. Investors in funds of hedge funds will incur asset-based fees and expenses at the fund level and indirect fees, expenses and asset-based compensation of investment funds in which these funds invest. An investment in a hedge fund involves the risks inherent in an investment in securities as well as specific risks associated with limited liquidity, the use of leverage, short sales, options, futures, derivative instruments, investments in non-U.S. securities, junk bonds and illiquid investments. There can be no assurances that a manager's strategy (hedging or otherwise) will be successful or that a manager will use these strategies with respect to all or any portion of a portfolio. Please carefully review the Private Placement Memorandum or other offering documents for complete information regarding terms, including all applicable fees, as well as other factors you should consider before investing.

Buying commodities allows for a source of diversification for those sophisticated persons who wish to add commodities to their portfolios and who are prepared to assume the risks inherent in the commodities market. Any purchase represents a transaction in a non-income producing commodity and is highly speculative. Therefore, commodities should not represent a significant portion of an individual's portfolio. Buying gold, silver, platinum and palladium allows for a source of diversification for those sophisticated persons who wish to add precious metals to their portfolios and who are prepared to assume the risks inherent in the bullion market. Any bullion or coin purchase represents a transaction in a non-income-producing commodity and is highly speculative. Therefore, precious metals should not represent a significant portion of an individual's portfolio.

Trading foreign exchange involves a high degree of risk. Exchange rates between foreign currencies change rapidly do to a wide range of economic, political and other conditions, exposing one to risk of exchange rate losses in addition to the inherent risk of loss from trading the underlying financial product. If one deposits funds in a currency to trade products denominated in a different currency, one's gains or losses on the underlying investment therefore may be affected by changes in the exchange rate between the currencies. If one is trading on margin, the impact of currency fluctuation on that person's gains or losses may be even greater.

Investments that are concentrated in a specific sector or industry increase their vulnerability to any single economic, political or regulatory development. This may result in greater price volatility.

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