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CHAPTER 18

Risk-Managing the Uncertainty in VaR Model Parameters[1]

Jason C. Hsu and Vitali Kalesnik

ABSTRACT

Managing risk successfully requires a detailed understanding of the distributions from which random shocks to asset prices are drawn. However, there is uncertainty in both the actual distribution of returns and the parameters characterizing the distribution. In this chapter, we focus on the uncertainty in estimating the distributional parameters and how this uncertainty impacts value at risk calculations. We illustrate some traditional (but naïve) methods for handling parameter uncertainty and show that these methods could often lead to poor risk management results. We then provide techniques for quantifying risk more accurately when distribution parameters are estimated with low precision or when there are disagreements over the parameter estimates.

[1] The authors would like to acknowledge Bryce Little, Vivek Vishwanathan, and Feifei Li for their assistance in completing this chapter.

THE SUBPRIME CRISIS OF 2008

The most familiar risk measure used by practitioners has been the standard deviation of portfolio returns, introduced by Harry Markowitz (1959) in his seminal work on portfolio selection. However, this measure has a considerable drawback: it treats extremely favorable realizations in the same way as the extremely adverse ones. To overcome this shortcoming, several downside risk measures have been introduced and adopted. Most notably, value at risk (VaR), which was popularized in the mid-1990s by J.P. Morgan's RiskMetrics, has become a universal risk management tool in the finance industry. For financial institutions complying with the European Capital Adequacy Directive (CAD) and Basel II Accords or funds seeking qualification under UCIT-III, VaR modeling and computation are not just best practices—they are required. However, like other financial innovations such as mean-variance portfolio optimization and option pricing, successful application of VaR depends on the quality of one's model parameter inputs.

In 2008, 10 years after the 1998 Russian–Asian financial crisis that triggered the collapse of Long-Term Capital Management (LTCM), the world again witnessed a global crisis that threatens to destabilize our capital markets and financial institutions. This new crisis was triggered by the U.S. subprime mortgage debacle and has already brought the collapse of Bear Stearns, the world's fifth largest investment bank. Other investment banks have been forced to recapitalize by issuing mixtures of debt and equity to sovereign wealth funds at distressed prices. The Abu Dhabi Investment Authority (ADIA) acquired a $7.5 billion stake in Citigroup. Singapore's Government Investment Corporation (GIC) invested $9.75 billion and $6.88 billion into UBS and Citigroup, respectively. The GIC's sister entity, Temasek, along with Korean Investment Corporation (KIC), infused a combined $11 billion into Merrill Lynch. Chinese Investment Corporation invested $5 billion in Morgan Stanley. Before the dust settles, poor management of subprime exposure may very well lay claim to more victims.

In the face of mounting subprime losses and the ensuing financial markets crisis, it appears that the finance industry's application of VaR is still far from adequate. At the writing of this chapter, global subprime related losses have surpassed $215 billion according to Japan's Financial Services Agency (FSA). J.P. Morgan Chase and Deutsche Bank estimate that global losses could ultimately reach between $300 billion and $400 billion. This estimate dwarfs the loss posted by the LTCM collapse, which resulted in a wealth destruction of $4.6 billion to investors and financial counterparties.

Additionally, the amplification of asset class correlations has prompted a liquidity crunch throughout credit markets. Unfavorable lending conditions spurred an unprecedented series of liquidity injections and policy interventions from both the European Central Bank and U.S. Federal Reserve banks. It is estimated that the European Central Bank provided $500 billion in liquidity since late 2007. The Federal Reserve, in addition to offering $200 billion in bailout loans and guaranteeing Bear Stearns' balance sheet, embarked on a series of interest rate cuts in an attempt to thaw frozen credit markets. It appears that extreme (tail) events continue to catch our financial institutions by surprise. Noted risk author Nassim Taleb's Black Swans seem, somehow, more frequent than data or conventional wisdom would indicate. However, we argue later in this chapter that the problem might not be due to inadequate modeling of unexpected events but rather might be due to the inappropriate treatment of disagreements in investment beliefs.

Why have things gone so very wrong again? Banks are required to perform VaR calculations to ensure capital adequacy as well as to manage balance sheet risk. However, VaR did not seem to help financial institutions manage their subprime exposure adequately. Many argue that banks may have been using incorrect probability distributions to model asset price risk. Particularly, the distributions used for computing VaR may not sufficiently capture the frequency of extreme shocks to asset prices (kurtosis) as well as the size of the extreme shocks (negative skew). In this chapter, we contend that financial institutions have become sufficiently sophisticated and educated about fat tail distributions in risk modeling. Advanced applications of VaR often involve discussions of fat tail distributions such as Levy or Cauchy distributions.

The progress made in VaR research with respect to extreme event risk modeling has been tremendous since the days of LTCM. There are numerous articles in practitioner journals [see Lucas (2000)] addressing the issue of fat tail distribution and their modeling with respect to VaR. Nassim Taleb, whose Black Swan analogy poetically illustrates our natural tendency to underestimate randomness—or rather, overestimate knowledge—has continued to alert the finance industry to tail event risk. We, as an industry have invested significant resources in applying non-normal and leptokurtotic distributions to model extreme loss events. So, what went awry?

In this chapter, we argue that the poor risk management is caused in large part by the failure to properly recognize and account for disagreements in investment beliefs. In the academic literature, this is known as uncertainty in asset return distribution parameters. A number of financial pundits, including Bill Gross of PIMCO, have warned us of the risk of the aggressive mortgage lending practices and the ensuing real estate speculation that prevailed from 2002 through 2007. The subprime problem is not a Black Swan in that regard. Some investors expected the aggressive subprime lending to lead to problems, while others did not. Certainly, ex ante, neither is 100 percent correct; how do we account for these diverging market views? In the VaR language, we need to adjust for the reality that we do not have perfect information regarding the mean and other moments of the asset return distribution.

The uncertainty regarding distribution parameters can often be very substantial when there is significant disagreement on return assumptions. For example, suppose members of an investment committee disagree about the forward-looking state of the stock market. Two members on the committee believe that a bear market is forthcoming and expect the market to yield a –20 percent return. The remaining three members believe a bull market will continue and expect a 20 percent return. How do we model this difference in investment beliefs? If we do not correctly model this parameter disagreement, but instead naïvely accept the estimate as determined by the majority rule or by some blended averaging, we would mismanage ex ante portfolio risk. Fine tuning the fat tail characteristics would not redress the problem sufficiently.

In this chapter we present a method that appropriately accounts for distributional uncertainty. We illustrate the technique with examples of mean, variance, and correlation uncertainty. Specifically, we compare VaR statistics generated from this approach against other standard (and more naïve) approaches. We show that properly quantifying mean and variance uncertainty leads to significant improvement in ex ante risk characterization of an investment portfolio.

PARAMETER UNCERTAINTY

In traditional VaR analysis, we assume that the parameters, such as the mean and variance, characterizing the probability distribution are known with perfect accuracy. This seems rather counterintuitive since we readily admit our inability to determine the exact probability distribution to model shocks to asset prices. Probability distributions like the Levy distribution, the Cauchy distribution, and other fat tail stable Paretian distributions have been considered for modeling asset returns, in addition to the classic lognormal distribution. Academics and practitioners have argued over the merit of these different modeling choices but have generally conceded that uncertainty exists in identifying the right distribution model. We argue that the uncertainty regarding the mean and covariance is likely far greater in most investment decision process.

What makes estimating the mean and covariance of the return distribution challenging for VaR applications is the short time horizon over which the parameter estimates must hold true. While investing is a long horizon endeavor, risk management is a necessarily a short horizon activity. One simply cannot ignore capital adequacy violations and margin calls because of the assumed effect of time diversification or a belief in long-term price mean reversion after substantial price decline. From a modeling perspective, this means we cannot rely on estimates derived from long horizon sample averages in the same way we use them to design a 10-year horizon strategic investment portfolio.

Given the empirical evidence supporting time-varying equity risk premium and stochastic market volatility, we need to take into account these shifting distributional parameters as well as our inability to accurately estimate them. Additionally, within an investment organization diverging but valid beliefs on the forward-looking state of the economy often coexist. This disagreement in investment beliefs, which represents uncertainty in the true distributional parameters, needs to be treated appropriately in VaR calculations. Lastly, it is important to note that parameter uncertainty is very different from uncertainty in the ex-post asset returns. Realized stock returns can differ substantially from its mean return, even if we are 100 percent certain about the equity mean return. This is the nature of stock return volatility—it is not related to the uncertainty over the mean of the stock return distribution. The classic VaR application addresses the risk arising from return volatility and handles it very successfully. Where it falls short is in accounting for the uncertainty in the mean estimate. We make this point clear in the example that follows, where we illustrate the effect of parameter uncertainty on risk management.

AN ILLUSTRATIVE EXAMPLE WITH MEAN UNCERTAINTY

We revisit the previous example where a five-member investment committee is split 3 to 2 on the outlook for U.S. financial sector return over the next 12 months. We rewind the clock back to July 2007 when two Bear Stearns hedge funds collapsed from subprime investment losses. Three members on the committee believe that the subprime problem would be isolated to a few banks and that the market has largely priced in the full impact from the subprime problem (this surprisingly was the conventional Street wisdom at the time). Two members on the committee believe that the collapse of the Bear hedge funds was the beginning of a system-wide financial crisis. The bullish camp believes that after the sharp price correction, the forward-looking return for the financial sector would be very positive and would average 20 percent. The bearish camp believes that the financial sector return would be substantially negative at –20 percent. For simplicity, both sides assume a volatility of 15 percent; we will examine the effect of volatility uncertainty in the next section. Note that we use exaggerated numbers in our example to create a more stark illustration.

In this example, we consider four different risk assessment scenarios. Traditionally, the risk manager would take as inputs the assumptions provided by the investment committee; however there can be various ways to interpret the committee's outlooks when characterizing the distribution.

1. By majority balloting, the process would produce an expected return of 20 percent (in this case the parameters are mode estimates).
2. Perhaps the committee members would compromise and take averages of their views on mean and standard deviation, which would lead to an expected return of 4 percent.

3. Perhaps the risk manager would like to manage against the worst-case scenario and assume an expected return of –20 percent.
4. Finally, the risk manager might consider modeling the uncertainty in the expected mean return explicitly.

For the sake of simplicity, we only consider lognormal assumptions in this example. The analysis can be extended to fat tail distributions with similar results. We first write down the return distribution under the four different scenarios.

1. Majority rule estimate: $\ln r_1 \sim N\,(20\%, 15\%)$
2. Blended average estimate: $\ln r_2 \sim N\,(4\%, 15\%$
3. Worst-case estimate: $\ln r_3 \sim N\,(-20\%, 15\%)$
4. Parameter uncertainty:

$$\ln r_4 \sim \begin{cases} prob = \dfrac{3}{5}, N(20\%,15\%) \\ prob = \dfrac{2}{5}, N(-20\%,15\%) \end{cases}$$

where $N\,(\mu,\sigma)$ is a normal distribution with mean and standard deviation (μ,σ) and where the probability density function is

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right), \text{ for } -\infty < x < \infty$$

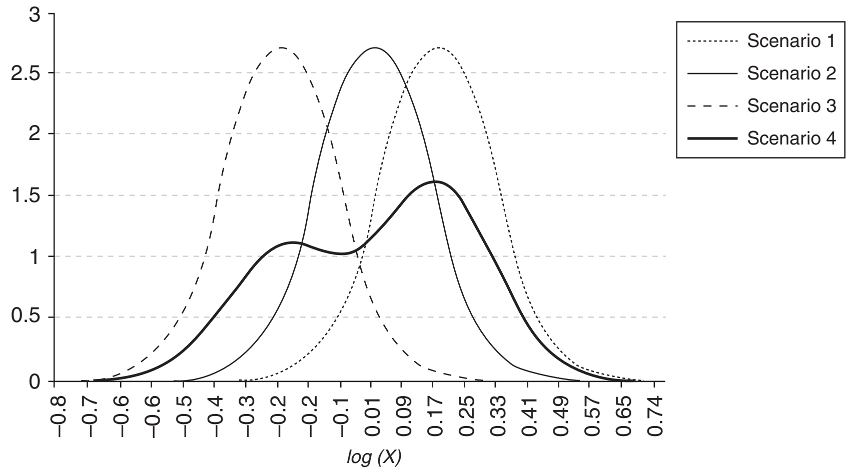
The density function for scenario 4 is therefore

$$g(x) = \frac{3}{5}f(x;20\%,15\%) + \frac{2}{5}f(x;-20\%,15\%), \text{ for } -\infty < x < \infty$$

We plot the four ex ante density functions in Figure 18.1. We also compute some basic risk statistics in Table 18.1. In the first four columns, we compute the first four moments for the different ex ante distributions for ln x. We then report, in column five, the VaR at 5 percent confidence assuming a portfolio that is 100 percent invested in the financial sector stocks. In

FIGURE 18.1

Probability Density Function of Investment with Mean Uncertainty



column six we report the expected percentage loss for this portfolio for outcomes in the negative 5 percent tail of the distribution. Finally, in column seven, we compute the maximum allowable portfolio allocation to financial stocks, in a stock-versus-cash portfolio, assuming a loss tolerance of –25 percent with 5 percent probability. This last statistic allows readers to compare portfolio allocations given identical loss tolerance assumptions. For example, a higher allocation to stocks in scenario 1 versus scenario 4 would suggest that the risk management assumptions in scenario 1 are less conservative.

Comparing scenario 1 versus scenario 2, we note that the driver of the disparity in VaR and associated risk statistics is the difference in the distribution mean assumption. Using the committee's blended view on the mean return estimate instead of the mode estimate (arrived from majority rule) leads to a more conservative risk estimation in this example. This suggests that a compromise in the committee's investment beliefs can have beneficial risk management properties relative to a majority rule approach for determining investment belief, when there is disagreement. However, the blended mean approach illustrated in scenario 2, while it represents an improvement over the majority rule approach, remains naïve and does not produce the correct risk management calculation. Observe that the density function in scenarios 2 and 4 have the identical distribution means at 4 percent. However, the standard deviation in scenario 4 becomes significantly larger when we correctly account for the uncertainty in the mean.

The uncertainty-adjusted model leads to a more accurate and more conservative risk assessment than the naïve model with a simple blended mean. Note that the uncertainty-adjusted distribution is no longer normal, which means parameter uncertainty can lead to a non-lognormal ex ante distribution assumption even when the underlying asset return process is lognormal.

Naturally, the worst-case scenario parameter assumed in scenario 3 leads to the most conservative risk management. However, this risk management approach is also not desirable, because it results in insufficient risk taking, which would hurt investment results. Note that the equity allocation, corresponding to a 5 percent chance of 25 percent loss, under the worst-case scenario, is 69 percent compared to 80 percent for the uncertainty-adjusted model. This represents a significant under-investment where as the majority rule and the blended estimates approaches lead to significant over-investment at 100 percent.

Observe that the log distribution with parameter uncertainty has a significantly higher variance and negative skewness which results in a starkly more conservative risk management guideline resulting from the VaR and expected shortfall calculation. Interestingly, we also observe a negative excess kurtosis in the distribution with uncertain mean. However, the negative excess kurtosis is entirely dominated by the

TABLE 18.1

Risk characteristics of investment with mean uncertainty

	Mean	**Volatility**	**Skewness**	**Kurtosis**	**VaR 5%**	**Expected Shortfall**	**Max % Invested (5% Chance of 25% Loss)**
Scenario 1	20.00%	15.00%	0.00%	0.00%	4.59%	10.42%	100.00%
Scenario 2	4.00%	15.00%	0.00%	0.00%	18.70%	23.70%	100.00%
Scenario 3	−20.00%	15.00%	0.00%	0.00%	36.05%	39.95%	69.36%
Scenario 4	4.00%	24.68%	−20.44%	−72.89%	31.13%	36.11%	80.30%
Scale	ln *x*	ln *x*	ln *x*	ln *x*	*x*	*x*	*x*

increase in the variance relative to the normal model with blended mean estimate. This suggests that capturing fat tails (or higher probability for extreme outcomes) may not be as important in risk management as capturing parameter uncertainty correctly.

AN ILLUSTRATIVE EXAMPLE WITH VARIANCE UNCERTAINTY

We now extend the previous example and consider a situation where there is uncertainty over the variance of the distribution. Suppose that the members on the investment committee agree on the forward-looking mean return for the financial sector. They expect return to be 10 percent but disagree on the volatility. Three members expect a forward environment with relatively modest volatility at 12 percent. The remaining two members expect a choppier market with volatility near the historical high of 25 percent.

Again, we consider four different risk assessment scenarios.

1. The majority rule process would produce a volatility assumption of 12 percent.
2. Blending the opinion of the committee would result in an estimated volatility of 17.2 percent.
3. The worst-case scenario assumes a volatility of 25 percent.
4. We apply the parameter uncertainty approach. The return distributions are
   a. Majority rule estimate: $\ln r_1 \sim N$ (10%, 12%)
   b. Blended average estimate: $\ln r_2 \sim N$ (10%, 17.2%)
   c. Worst case estimate: $\ln r_3 \sim N$ (10%, 25%)
   d. Parameter uncertainty: $\ln r_4 \sim \begin{cases} prob = \frac{3}{5}, N(10\%,12\%) \\ prob = \frac{2}{5}, N(10\%,25\%) \end{cases}$

where again $N(\mu, \sigma)$ is a normal distribution with mean and standard deviation (μ, σ) and where the probability density function is

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right), \text{ for } -\infty < x < \infty$$

The density function for scenario 4 is

$$g(x) = \frac{3}{5} f(x;10\%,12\%) + \frac{2}{5} f(x;-10\%,25\%), \text{ for } -\infty < x < \infty$$

We plot the ex ante density functions in Figure 18.2 and the basic risk statistics in Table 18.2.

Using the committee's blended view on the volatility instead of using the majority rule approach leads again to a more conservative risk assessment. For the density function in scenarios 2 and 4, again, the standard deviation in scenario 4 becomes larger when we correctly account for the uncertainty in variance. Note, however, that the increase in standard deviation was not as pronounced as the situation when there is uncertainty in the distribution mean. Note also that in scenario 4 where we adjust for variance uncertainty, kurtosis becomes positive. The increase in volatility and the increase in kurtosis both contribute to a more conservative risk assessment than the naïve model with a simple blended standard deviation.

Observe that when we adjust for variance uncertainty appropriately, the resulting ex ante log distribution has a slightly higher variance and does not show a negative skew as was seen in the ex ante distribution with

FIGURE 18.2

Probability Density Function of Investment with Variance Uncertainty

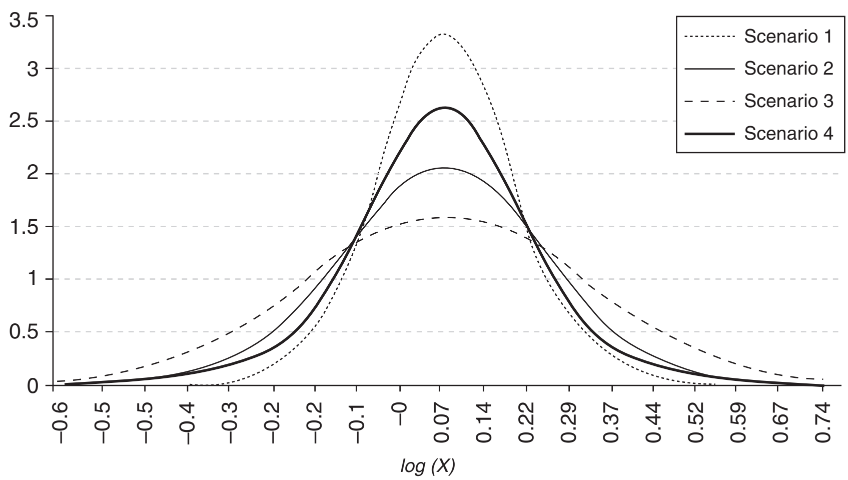


TABLE 18.2

Risk characteristics of investment with variance uncertainty

	Mean	Volatility	Skewness	Kurtosis	VaR 5%	Expected Shortfall	Max % Invested (5% Chance of 25% Loss)
Scenario 1	10.00%	12.00%	0.00%	0.00%	9.31%	13.93%	100.00%
Scenario 2	10.00%	17.20%	0.00%	0.00%	16.74%	22.40%	100.00%
Scenario 3	10.00%	25.00%	0.00%	0.00%	26.77%	33.87%	93.38%
Scenario 4	10.00%	18.34%	0.00%	147.20%	18.09%	26.68%	100.00%
Scale	ln *x*	ln *x*	ln *x*	ln *x*	*x*	*x*	*x*

mean uncertainty. However, we do pick up positive kurtosis in the face of variance uncertainty. The uncertainty in the variance estimate transforms the lognormal distribution into a fat tail distribution. It is this increase in kurtosis that drives much of the disparity in risk assessment between the blended average approach and the uncertainty approach. Note that when there is uncertainty in the variance estimate, the uncertainty approach can lead to similar risk assessment outcome as an approach that assumes a fat tail distribution.

AN ILLUSTRATIVE EXAMPLE WITH CORRELATION UNCERTAINTY

We extend the above example to study the effect of correlation uncertainty in a too risky asset environment. We consider investments in U.S financial stocks and in commodities. For simplicity, suppose the two asset classes will have equal weights in the portfolio, and the aforementioned committee members agree on both the mean and the variance of the bivariate distribution and disagree only on the correlation. For simplicity, we assume the vector of means and standard deviations are $\mu = (10\%, 10\%)$ and $\sigma = (12\%, 12\%)$. Suppose three members have a view that stocks and commodities would have a negative correlation of –30 percent; they assume that commodities exposure is a good hedge against equity risk. Suppose the remaining two members believe that the forthcoming

U.S. recession would suggest temporary lower demand for commodities, which suggest that equity prices and commodity prices would become correlated on the downside, making the commodity investment a poor hedge; they assume a 90 percent short-term correlation.

In this example we compare only the blended average approach represented in scenario 1 with the parameter uncertainty approach in scenario 2. We write down the joint distribution density function of the form $N(\mu_x, \mu_y; \sigma_x, \sigma_y; \rho)$. Again, for simplicity and with no loss of generality, the log returns for scenario 1 are assumed to be bivariate normal.

1. Blended average: $(\ln r_1, \ln r_2) \sim N(10\%, 10\%; 25\%, 25\%\ 18\%)$, where the joint density function is

$$f(x,y;\mu_x,\mu_y,\sigma_x,\sigma_y,\rho)$$
$$= \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left(\frac{-1}{2\left(1-\rho^2\right)}\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}-\frac{2\rho xy}{\sigma_x\sigma_y}\right)\right), \text{ for } -\infty < x,y < \infty$$

The density function for parameter uncertainty is then

2. Correlation uncertainty:

$$g(x,y) = \frac{3}{5} f(x,y;10\%,10\%;25\%,25\%,-30\%)$$
$$+ \frac{2}{5} f(x,y;10\%,10\%;25\%,25\%;90\%), \text{ for } -\infty < x,y < \infty$$

Since the allocations to equities and commodities are fixed at 50 percent each, we can derive the density function for the portfolio log return from the joint density function by integrating over x and y with the constraint that the portfolio return $r = 0.5x + 0.5y$. Using the portfolio return density function we can compute the portfolio VaR. We plot the two portfolio return density functions in Figure 18.3 and present the risk statistics in Table 18.3. In both scenarios, portfolios have identical mean return and volatility. Properly accounting for the correlation uncertainty, results in a significantly fatter tail, as seen by the large excess kurtosis. The excess kurtosis means that the portfolio risk appetite falls dramatically, even with similar mean and variance.

FIGURE 18.3

Probability Density Function of Investment with Correlation Uncertainty

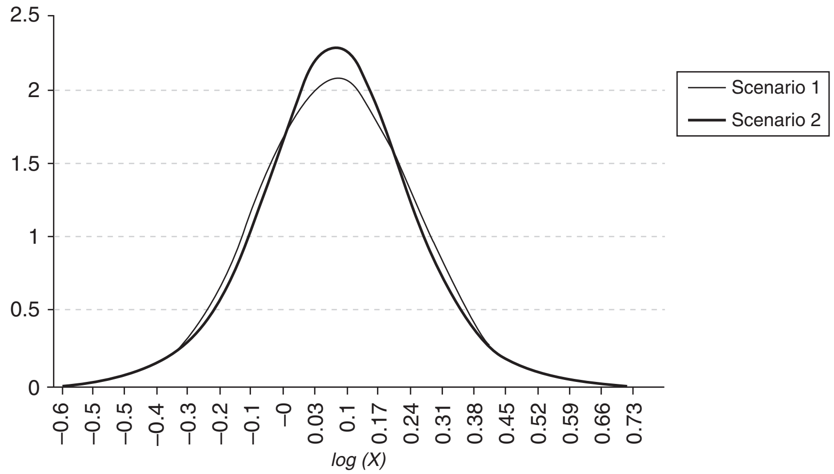


TABLE 18.3

Risk characteristics of investment with correlation uncertainty

	Mean	Volatility	Skewness	Kurtosis	VaR 5%	Expected Shortfall	Max % Invested (5% Chance of 25% Loss)
Scenario 1	10.00%	19.20%	0.00%	0.00%	29.31%	33.69%	85.29%
Scenario 2	10.00%	19.20%	0.00%	74.46%	31.73%	37.58%	78.79%
Scale	ln x	ln x	ln x	ln x	ln *x*	ln *x*	ln *x*

CONCLUSION

In this chapter, we present a technique that appropriately handles disagreements in investment belief or uncertainty in return distribution parameters. Disagreements in investment belief are common in a diverse and healthy investment organization. People with different experiences and perspectives will often have different investment outlooks. Members on the investment committee will disagree on the outlook for asset class returns, on the forward-looking volatility as well as correlations across the asset classes. From a modeling standpoint, diverse investment outlooks can be characterized as uncertainty about the parameters that govern the joint distribution of asset returns. This is a far stronger statement regarding our inability to forecast the future than what is assumed in standard models. Not only are we unable to forecast the random shocks to the economy that result in volatility in asset returns, we are actually unable to characterize the random distribution which governs the asset returns with certainty. In other words, we are uncertain about the parameters of the probability distribution from which the random returns are drawn.

However, existing standard risk management approaches do not properly handle parameter uncertainty. This, we believe has led to inadequate risk management, which we believe has led to some systemic crises in the financial industry, despite the widespread application of VaR systems. The U.S. subprime crisis, which, according to Street estimates, will ultimately create more than $300 billion in losses for global financial institutions, has again brought to focus the failure of our current risk management practices. It may be convenient to argue that the subprime crisis was a six-sigma event or a Black Swan event that could only be modeled with the most sophisticated fat tail distributions. We posit, however, that the problem may occur with the inappropriate modeling of parameter uncertainty. We illustrate, with a few simplified examples, where the traditional methods for estimating distribution parameters lead to suboptimal risk management when parameters are uncertain. The resulting risk statistics often understate the true risk. Specifically, if beliefs regarding mean and covariance were created through a majority rule process, where the most popular estimates were selected, we would find suboptimal risk taking relative to the proper ex ante belief distribution. The resulting risk characteristics would either wildly under- or overestimate the true risk. If we use a blended average approach to reach a compromise estimate on the mean and covariance, the resulting risk characteristics would always underestimate the true risk and often quite substantially. We show additionally, that using a fat tail distribution to account for potential extreme events does not produce the same risk management effect as accounting for parameter uncertainty and often still results in understating the true risk.

In conclusion, while rare extreme events may contribute to the crises in our financial markets, it is more likely that our risk management approach has simply not accounted for parameter uncertainty appropriately. The effect will be particularly severe in situations where the investment beliefs are very diverse, reflecting large uncertainty in the return distribution parameters. Imagine the debates that went on at the major investment banks as executives argued over the wisdom of holding subprime mortgage papers as triple-A collaterals. There was likely a minority group of executives who forecasted a decline in real estate prices, which would suggest a significantly negative expected return on the subprime mortgage papers. Ultimately, this view was not supported by the majority opinion or had led to only a small revision downward in return assumptions on the subprime papers; this meant that the resulting VaR statistics would understate the ex ante risk. We suspect that had the uncertainty been appropriately modeled, the VaR calculations would have produced very different risk statistics, which might have led the banks to reduce their exposure to subprime related instruments.

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