Prospect Theory and Its Applications in Finance

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Prospect theory is an important theory for decision making under uncertainty. It departs from the traditional expected utility framework in important ways. It provides psychological underpinnings for the behavioral approaches to portfolio selection that are quite different from the traditional approaches such as the mean-variance framework. Prospect theory was developed by two psychologist, Daniel Kahneman and Amos Tversky, and published in the Econometrica in 1979. Kahneman won the 2002 Nobel Prize for Economics "for having integrated insights from psychological research into economic science, especially concerning human judgment and decision-making under uncertainty."[1] The 1979 paper on prospect theory was singled out for praise by the Royal Swedish Academy of Science.

Since its first appearance, prospect theory has been revised and improved in many ways (e.g., Tversky and Kahneman, 1992, Wakker and Amos Tversky, 1993, Wakker and Zank, 2002). It has also been widely applied to many areas of social science. This paper reviews the prospect theory and its applications in finance.

1 What is Prospect Theory?

The traditional finance theory assumes that investors make decision under uncertainty by maximizing expected utility of wealth or consumption. The expected utility theory is mathematically elegant and is a rational-based framework built upon axioms. However, the underlying assumptions have been shown by many studies to be inaccurate description of how people actually behave when choosing among risky alternatives.[2]

Kahneman and Tversky (1979) propose prospect theory as a descriptive model of decision making under uncertainty. The prospect theory is NOT a normative theory, but a descriptive approach to explain real world behavior. Kahneman and Tversky relied on a series of small experiments to identify the manner in which people make choices in the face of risk.

Like its mean-variance theory counterpart from the traditional approach, prospect theory focuses on the way people choose among alternatives. But the theories are

[1] Tversky passed away in 1996.

[2] For example, the famous Allias paradox show that the independence axiom of expected utility theory is routinely violated in the real life.

different. People who conform to prospect theory tend to violate the principles that underlie mean-variance theory.

1.1 Key Elements of Prospect Theory

There are four ingredients in prospect theory that distinguishes it from mean-variance theory. First, people in mean-variance theory choose among alternatives based on the effect of the outcomes on the levels of their wealth. In contrast, people in prospect theory choose based on the effect of outcomes on changes in their wealth, relative to their reference point. In other words, prospect theory agents evaluate outcomes in terms of gains and losses relative to a reference point.[3]

Second, people in mean-variance theory are risk averse in all of their choices. In contrast, prospect theory agents are risk-averse in the domain of gains but risk-seeking when all changes in wealth are perceived as losses. Consider the following experiment that illuminates the features of prospect theory. Imagine that you face a concurrent choice within two pairs (A vs. B and C vs. D), where:

- A = a sure gain of \$24,000
- B = a 25% chance to gain \$100,000 and a 75% chance to gain nothing.
- C = a sure loss of \$75,000
- D = 75% chance to lose \$100,000 and 25% chance to lose nothing.

Kahneman and Tversky found that more people chose A than B and more people chose D than C. This common choice is a puzzle if agents are always risk-averse and never risk-seeking. While the choice of A over B is consistent with risk aversion, the choice of D over C is not. Instead it is consistent with risk-seeking. Note that the \$25,000 expected gain of B (25% of a \$100,000 gain), is greater than the sure \$24,000 gain of A, so the common choice of A over B is consistent with risk-aversion. However, the common choice of D over C indicates that most people make some choices as if

[3]This aspect of prospect theory is similar to habit-formation or catching-up-with-Jones utility function where agents care about their consumption relative to some benchmark levels (e.g., their neighbor's consumption, or a certain level of consumption they have been used to have). Other aspects of prospect theory clearly are different from the habit-formation utility.

they are risk-seeking. Note that the expected $75,000 loss of D (75% of a $100,000 loss) is equal to the sure $75,000 loss of C, but D is riskier than C since it can impose a $100,000 loss. Kahneman and Tversky refers to the choice of D over C as "aversion to a sure loss," since C imposes a sure loss while D does not. An individual who has not made peace with his losses is likely to accept gambles that would be unacceptable otherwise

The third feature of prospect theory is often called "loss aversion." An individual is loss averse if she or he dislikes symmetric 50-50 bets and, moreover, the aversiveness to such bets increases with the absolute size of the stakes. In other words, there is an asymmetry in how prospect agent perceive gains and losses of equal amounts. Loss aversion says that the disutility of giving up a valued good is much higher than the utility gain associated with receiving the same good.[4] Loss aversion applies when one is avoiding a loss even if it means accepting a higher risk. Some argue that investors and traders show no risk aversion, but an aversion against losses.

The concept of loss aversion can be illustrated by an example in Samuelson (1963). Samuelson once offered a colleague the following bet: flip a coin, heads you win $200 and tails you lose $100. Samuelson reports that his colleague turned this bet down: "I won't bet because I would feel the $100 loss more than the $200 gain." This sentiment is the intuition behind the concept of loss aversion.

Finally, people in mean-variance theory treat risk objectively, by its probabilities. In contrast, the utility of prospect theory agent depends not on the original probability but rather on the transformed probability. These transformed probabilities can be viewed as decision weights, or subjective probabilities. They do not just measure the perceived likelihood of an event. Instead, they measure the impact of events on the desirability of prospects. This features of the prosect theory can explain several key violations of expected utility theory, including the famous Allais' parodox. People in prospect theory overweight small probabilities. Overweighting small probabilities explains people's demand for lotteries offering a small chance of large gain, and for insurance protecting against a small chance of a large loss.

To summarize, under prospect theory, people evaluate risk using a value function

[4]This is sometimes called endowment effect: the value appears to change when a good is incorporated into one's endowment. For example, there is a large difference in willingness to pay and willingness to accept, even if the the sellers where just equipped with the good.

that is defined over gains and losses, is concave over gains and convex over losses, and is kinked at the origin; and using transformed rather than objective probabilities by applying a weighting function.

1.2 Prospect Theory Value Function

Let us illustrate how prospect theory agent evaluates risk, and how the four elements of the prospect theory can be reflected in the value or utility function. Consider a simple gamble that with probability p, you get x and with probability q, you receive y, where $x < 0 < y$, and $p + q = 1$. In the expected utility framework, the agent evaluate this risk by computing

$$p\ U(W + x) + q\ U(W + y)$$

where W is agent's current wealth. Under the prospect theory of Kahneman and Tversky (1979), the agent assigns the gamble the value

$$\pi(p)v(x) + \pi(q)v(y)$$

where π is a probability transformation, and v is an S-shaped prospect theory value function like that shown in Figure 1.

Loss aversion implies a kink in the prospect theory value function around the reference point (the "origin"), with the slope being steeper for losses than for gains. The decline in utility for a loss (measured relative to a reference point) exceeds the increase in utility for an equal-sized gain (relative to the same reference point). Kahneman and Tversky (1979) infer the kink in utility from the widespread aversion to bet that payoffs \$110 when a head comes up with a coin flip and -\$100 when a tail come up. Such aversion is hard to explain with differentiable utility function, because the very high local risk aversion required to do so typically predicts implausibly high aversion to large-scale gambles, such as 50:50 bet to win \$20 million or lose \$10,000, which clearly is not reasonable (e.g., Epstein and Zin, 1990, Rabin, 2000, Barberis, Huang and Thaler, 2003). Extensive experiments reveal that people are generally about twice as unhappy about a given loss as the joy brought by a gain of the same size. For example, the disutility of lossing \$100 is roughly twice the utility of gaining

$100.

A typical prospect theory value function is given by the following:

$$\begin{aligned} v(W) &= \frac{(W-R)^{1-\gamma}}{1-\gamma}, \qquad \text{if } W \geq R; \\ v(W) &= -\lambda \frac{(R-W)^{1-\gamma}}{1-\gamma}, \quad \text{if } W < R \end{aligned}$$

where R be a reference level, γ is a positive constant, and $\lambda > 1$ is another constant. In figure 1, $\gamma = 0.5$ and $\lambda = 2.25$. Function $v(W)$ is continuous and differentiable (except at $W = R$). It is S-shaped (see Figure 1): it is monotonic increasing; it is concave in the region $W > R$ and convex in the region $W < R$. The S-shaped value function can also be generated by the following function:

$$\begin{aligned} v(W) &= 1 - e^{-\gamma(W-R)}, \qquad \text{if } W \geq R; \\ v(W) &= \lambda(e^{-\gamma(R-W)} - 1), \quad \text{if } W < R \end{aligned}$$

1.3 Cumulative Prospect Theory

Perhaps the most important change to the original prospect theory is that of Tversky and Kahneman (1992) about how probabilities are transformed. The original specification in Kahneman and Tversky (1979) applies only to binomial gambles, and violates the first-order stochastic dominance property. The essence of change in Tversky and Kahneman (1992) is that the transformation is first applied to the cumulative density function rather than directly to the probabilities. Thus, the Tversky and Kahneman (1992) version is usually called the cumulative prospect theory. The cumulative prospect theory applies to the most general gambles. It is also consistent with first-order stochastic dominance.

More precisely, cumulative prospect theory says that the agent evaluates a gamble that pays

$$x_{-m} < x_{-m+1} < \cdots < x_{-1} < x_0 = 0 < x_1 < \cdots < x_n$$

with corresponding probabilities $p_{-m}, \cdot, p_{-1}, p_0, p_1, \cdots, p_n$ by assigning it a value

$$\sum_{i=-m}^{n} \pi_i \; v(x_i)$$

where v is a S-shaped value function as above, and

$$\begin{aligned} \pi_i &= w^+(p_i + \ldots + p_n) - w^+(p_{i+1} + \ldots + p_n) \qquad \text{if } 0 \le i \le n; \\ \pi_i &= w^-(p_{-m} + \ldots + p_i) - w^-(p_{-m} + \ldots + p_{i-1}), \quad \text{if } -m \le i \le 0 \end{aligned}$$

with w^+ and w^- being the probability weighting function for gains and losses respectively. The most common probability weighting function is given by

$$w^+(p) = w^-(p) = \frac{p^\alpha}{(p^\alpha + (1-p)^\alpha)^{1/\alpha}}, \qquad 0 < \alpha \le 1$$

1.4 Prospect Theory and Framing

Before we turn to the applications of prospect theory in finance, it is worth emphasizing that the framing of alternatives affect choices in the prospect theory, and people are assumed to be passive in accepting the frames or problem descriptions offered to them. In contrast, the traditional theory implicitly assumes that frames does not affect choice. For example, in the example described before (see Page 2), the common choice of the A & D combination is stochastically dominated by the less frequently chosen B & C combination. Note that A & D offers a 25% chance to win \$24,000 and a 75% chance to lose \$76,000, while B & C offers a 25% chance to win \$1,000 more and a 75% chance to lose \$1,000 less. Even though the Kahneman and Tversky's experiment instructions indicate that the choice among A, B, C and D is concurrent, people tend to frame the choice into one from A and B and one from C or D, overlooking the link between the two choices and its relationship to the final level of their wealth. This has important implication for investors' portfolio choices, which we will discuss in detail later.

Prospect theory also implies a unique relationship of risk taking to positive and negative framing. Negatively framed problems decrease risk bearing and encourage risk seeking. Since losses loom larger than gains, it appears that humans follow

conservative strategies when presented with a positively-framed dilemma, and risky strategies when presented with negatively-framed ones. To illustrate, consider the experiment in Kahneman and Tversky (1984) where they asked a representative sample of physicians the following question.

> Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences of the programs are as follows: If program A is adopted, 200 people will be saved. If program B is adopted, there is a one-third probability that 600 people will be saved and a two-thirds probability that no people will be saved. Which of the two programs would you favor?

Notice that the preceding dilemma is positively framed. It views the dilemma in terms of "lives saved." When the question was framed in this manner, 72% of physicians chose A, the safe-and-sure strategy, but only 28% chose program B, the risky strategy. An equivalent set of physicians considered the same dilemma, but with the question framed negatively:

> Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences of the programs are as follows: If program C is adopted, 400 people will die. If program D is adopted, there is a one-third probability that nobody will die and a two-thirds probability that 600 people will die. Which of the two programs would you favor?

The two questions examine an identical dilemma. Two hundred of 600 people saved is the same as 400 of 600 lost. However, when the question was framed negatively, and physicians were concentrating on losses rather than gains, they voted in a dramatically different fashion. When framed negatively, 22% of the physicians voted for the conservative strategy and 72% of them opted for the risky strategy! Clearly, framing can powerfully influence the way a problem is perceived, which in turn can lead to the favoring of radically different solutions.

Related to framing, psychologists have long known of the existence of the "positivity bias," which states that humans overwhelmingly expect good things (as opposed to neutral or bad things) to occur. If perceivers construct a world in which primarily positive elements are expected, then negative information becomes perceptually salient. We also know that people stop to examine disconfirmations to a much higher degree than confirmations.[5] Negative information is often highly informative and thus may be assigned extra weight in the decision-making process. This may have important implications for investors' reaction for good v.s. bad earnings or analyst recommendations.

2 Applications of Prospect Theory

Two types of applications are emphasized. Prospect theory can help us understand the portfolio choices and trading behavior of both individual investors and money managers in the financial market, such as why many investors tend to hold on to their losers and why they hold very undiversified portfolios. Prospect theory has also been applied to explain almost all well-known asset pricing "anomalies" including the equity premium puzzle, the profitability of value and momentum strategy, excess volatility, IPO underpricing and long-term performance of IPOs. Each of these applications will be discussed in this section.

2.1 Portfolio Choice

Expected utility theory implies that investors hold well diversified portfolios, vary their risk exposure by selecting the right mix of the risk free security and a risky fund that is itself well diversified. By way of contrast, prospect theory implies that investors do not choose well-diversified portfolios. In particular, people ignore covariance among security returns and therefore, choose stochastically dominated portfolios that lie below the efficient frontier. They also combine very safe and very risky choices in their portfolios (insurance and lottery tickets).

[5]By not looking, you won't find anything that may cause you to worry. If you look, you may find something you don't want to find!

Shefrin and Statman (2000) explain that behavioral portfolios of securities are structured as separate layers of a pyramid, e.g., a "downside protection" layer and an "upside potential" layer. For example, in the case of pension fund portfolios, the line between the downside protection and upside potential layers is the full funding line. The downside protection layer contains assets needed for full funding of pension obligations. The upside potential layer contains assets beyond those necessary for full funding. For individual investors, they may have a low aspiration layer that is designed to avoid poverty as well as a high aspiration layer for a shot at riches.

Prospect theory affects the contents of layers in the pyramid of behavioral portfolios. The parameters that are relevant to asset allocation in the behavioral portfolio theory are the relative importance of the upside potential goal relative to the downside protection goal and the reference points of the upside and downside goals. The greater importance that the investor attaches to the upside potential goal, the higher is his allocation of his wealth to the upside potential layer. At the same time, a higher reference point for the upside potential layer will be accompanied by the selection of securities that are more "speculative." The portfolios of prospect theory investors are sensitive to the location of the reference point. For low reference points, prospect theory investors choose traditional portfolios. Higher reference points induce risk-seeking behavior, or the reluctance to engage in trade.

The S-shape of the utility function also matters for behavioral portfolio choice. Higher concavity in the domain of gains reflects earlier satiation with a given security, and early satiation leads to an increase in the number of securities in a layer. Another determining factor is the degree of aversion to realization of losses. Investors who are aware of their aversion to the realization of losses hold more cash so as to avoid the need to satisfy liquidity needs by realization of losses. Moreover, portfolios of such investors contain securities held solely because selling them entails the realization of losses. These portfolios might *seem* well diversified, but the large number of securities they contain is designed for avoiding the realization of losses, not the benefit of diversification.

2.2 Disposition Effect

This subsection discusses investors' reluctance to realize losses in more detail. Shefrin and Statman (1985) coin the term "the disposition effect" to describe the tendency of investors to hold onto their losing stocks to a greater extent than they hold onto their winners. The disposition effect has been documented for both individual and professional investors in many markets and for many countries.

For example, Odean (1998) analyzes accounts at a large brokerage house and found that there was a greater tendency to sell stocks with paper capital gains than those with paper losses. Grinblatt and Keloharju (2001) find a similar effect among all types of investors in Finland, even after controlling for a variety of variables that may determine trading. Heath, Huddart and Lang (1999) uncover disposition behavior relative to a reference price of a prior high for the stock price by studying the option exercise behavior of over 50,000 employees at seven corporations. Shapira and Venezia (2001) show that both professional and independent investors in Israel exhibit the disposition effect, although the effect is stronger for independent investors. Garvey and Murphy (2004) provide evidence that even professional traders are also subject to the disposition effect. Locke and Mann (1999) present evidence for the existence of a disposition effect within a sample of professional futures traders. Frazzini (2004) and Wermers (2003) find evidence for disposition effect among mutual fund managers. The disposition effect also influences agents in the IPO and housing markets (e.g., Case and Shiller, 1988, Genesove and Mayer, 2001, Kaustia, 2004)

The disposition effect can be explained by the prospect theory, combined with the concept of mental accounting (e.g., Thaler, 1985). Mental accounting provides a foundation for the way that decision makers set reference points for the accounts that determine gains and losses. In the context of financial transactions, the key mental accounting issues concern aggregation: how transactions are grouped both cross-sectionally (e.g., are securities evaluated one at a time or as portfolios) and intertemporally (how often are portfolios evaluated). The main idea of mental accounting is that decision makers tend to segregate different types of gambles into separate accounts, and then apply prospect theory to each account by ignoring possible interactions.

The key feature of prospect theory required to explain disposition effect is con-

vexity over losses (the S-shaped value function). Intuitively, when an investor first bought a stock, he opened a mental account and keeps a running score on the gain or loss for his position. Suppose the investor suffers a paper loss. If he sells the stock and realizes the loss, the mental account is closed and the loss becomes certain. This is very painful for a prospect theory agent because of loss aversion and will inflict him trying to avoid such a loss. In fact, because he is risk-seeking in a losing situation, he will take even greater position ("throw good money after bad") in hope that prices will recover so he can break even sooner. On the other hand, in a winning situation the circumstances are reversed. Investors will become risk averse and quickly take profits, not letting profits run.

To demonstrate disposition effect using prospect theory utility function, consider Figure 1, which plots the S-shaped value function for outcomes in a particular stock. Let us analyze how this S-shape alters traditional investment behavior. The curve above the point labelled "reference point" has the shape of power utility. For true power utility, the fraction of wealth invested in the stock is increasing in the stock's expected return, but is unaffected by the (initial wealth) starting point. How is this demand function shifted by the substitution of a convex utility function to the left of the inflection point? Comparing a starting position at Point D with Point C in Figure 1, one can infer that demand is increased more at Point C. If we start from Point D, gambles rarely end up in the convex portion of the curve. Indeed, for any given positive mean return, demand increases as the starting position moves left of point D because gambles experience an increasing likelihood of outcomes in the convex portion of the value function. This pattern of larger demand (for a given mean) as the starting position moves left continues as our starting position crosses the inflection point and moves into the convex region. Clearly, the critical determinant of demand is the starting position in the value function.

When the relevant mental accounts employ the cost basis in a stock as the reference point, the starting positions are dictated by the unrealized capital gain or loss in the stock. Stocks that are extreme winners start the investor at Point D. Stocks that are extreme losers start the investor at Point A, and so forth. It follows that a PT/MA demand function differs from that of a standard utility investor not just because winners are less desirable than losers, other things equal. One also concludes that there is a greater appetite for large losers (point A) than for small losers (point B).

Moreover, there is a lesser desire to shun small winners (point C) than large winners (point D) because of the greater degree to which realizations in the convex region enter the expected value calculation.

2.3 Home Bias

Under the mean-variance framework, one of the key alleged benefits of international diversification is the minimization of risk for a given expected return. This contradicts the so-called "home bias", documented by French and Poterba (1991) and many others. The home bias refers to the finding that American investors hold more U.S. stocks and fewer foreign stocks than the amounts predicted by mean-variance optimization. It is a puzzle within the mean-variance framework but is is consistent with behavioral portfolio theory.

Stracca (2002) argues that if prospect theory is an accurate description of human attitudes towards risk, the benefits of international diversification would be reduced to a significant extent. He shows that risk concentration ("do put your eggs in the same basket") may be optimal for a cumulative prospect theory agent, provided that the subjective probability of obtaining a perfect hedge is negligible, and the agent sees the allocation of risks as a self-contained decision problem. The intuitive reason behind this result is that a prospect theory agent is risk-seeking over losses, with the consequence that the property of diversification of averaging downside risks is welfare-reducing rather than welfare-improving. In other words, risk diversification does not lead to risk (loss) minimization for cumulative prospect theory agent, in contrast with a standard expected utility agent.

While prospect theory can explain the tendency to concentrate risks on a single asset rather than to hold a well diversified portfolio, it can not explain why the single asset the investor chooses is a domestic one. One reason for this could be a greater familiarity with domestic assets. Consider a foreign stock and a domestic stock with an identical distribution of payoffs. Since foreign stocks seem less familiar than domestic stocks, investors may perceive it as having higher variance of payoffs. That perception leads to a low allocation to foreign stocks. A direct implication is a behavioral portfolio theory prediction that the home bias would decline as investors became more familiar with foreign stocks. There is no such prediction in mean-

variance portfolio theory.

2.4 Equity Premium

Mehra and Prescott (1985) discovered the equity premium puzzle. The puzzle is that the historic equity premium has been very large. Over the time period Mehra and Prescott studied (1889-1978) the annual real return on the S&P 500 was about 7 percent while the return on T-bills was less than 1 percent. Since 1978, stocks have done even better. When these large return differences are cumulated, the differentials become staggering. For example, a dollar invested in the S&P 500 on January 1, 1926, was worth over $1100 by the end of 1995, while a dollar invested in T-bills was worth only $12.87. Mehra and Prescott show that it is difficult to explain the combination of a high equity premium and a low risk-free rate within a standard neoclassical model. The implicit coefficient of relative risk aversion needed to produce such numbers was over 30, while most estimates put it close to 1.

Benartzi and Thaler (1995) offer an explanation of the equity premium based on the prospect theory. The key feature of prospect theory used in their explanation is loss aversion. They also need two closely related behavioral concepts: mental accounting and narrow framing. A financial investor can be modelled as making a series of decisions about the allocation of his assets. Mental accounting determines both the framing of decisions and the experience of the outcomes of these decisions. An investor who frames decisions narrowly will tend to make short-term choices rather than adopt long-term policies. An investor who frames past outcomes narrowly will evaluate his gains and losses frequently. In general, narrow framing of decisions and narrow framing of outcomes tend to go together, and the combination of both tendencies defines a myopic investor.

Benartzi and Thaler (1995) examine the effect of myopic loss aversion on risk attitudes and equity premium. They argue that the attractiveness of the risky asset depends on the time horizon of the investor, and valuation depends on investors' time horizon. Since the stock price is generally the frame of reference, the probability of loss or gain is important. Note that the more frequently one evaluate his portfolio, the more likely he sees losses and hence suffer from loss aversion. The probability of a gain or a loss for a risky asset that pays an expected 7 percent per year with a

standard deviation of 20 percent (like stocks) in the very short term is close to 50-50. To a loss-averse investor who evaluate his portfolio frequently, the stock market appears very risky. On the other hand, an investor who is prepared to wait a long time before evaluating the outcome of the investment as a gain or a loss will find the risky asset more attractive, since the longer the holding period, the higher the probability that he ends up with a positive total return.

Thus, if losses cause more mental anguish than equivalent gains cause pleasure, the experienced utility associated with owning stocks is lower for the more myopic investor. Long-term investors (individuals who evaluate their portfolios infrequently) are willing to pay more for an identical risky asset than short-term investors (frequent evaluation). The more often that a loss-averse investor evaluates his portfolio, the higher return he would demand in order to hold risky stocks. Benartzi and Thaler (1995) find that investors with myopic loss aversion would be indifferent between the historical returns of stocks and T-bills if they evaluate their portfolios about once per year.

2.5 Aggregate Stock Market Return Puzzles

Besides the equity premium puzzle, prospect theory can help explain other puzzling features of the aggregate stock market. Here we discuss two such examples: the volatility puzzle, and the predictability of price-earnings ratio.

The volatility puzzle is that stock market levels appear to move around too much. For example, ratios of price to earnings in the U.S. stock market have often been very high. The standard rationalization of this is that investors must be expecting high cashflows and earnings in the future, and are therefore happy to pay high prices today. However, historical data shows that high levels of price-earnings ratios are not, on average, followed by higher earnings. In this sense, it is a puzzle why prices were so high to begin with.

Historical data also shows that the price-earnings ratio can predict future returns on the stock market. High levels of the price-earnings ratio have generally led to lower subsequent returns, and low levels of the ratio to higher returns.

Barberis, Huang and Santos (2001) show that prospect theory help explain both

puzzles above. They need two features from prospect theory: agents receive direct utility from changes in the value of their financial wealth and they are loss averse. Barberis, Huang and Santos (2001) do not need the convexity over losses or the probability weighting features of the prospect theory.

However, they need to combine prospect theory, originally developed for one-shot game, with the "house money" effect of Thaler and Johnson (1990) which describe how people integrate gains and losses in dynamic setting. The house money effect simply says that people are more likely to bet recklessly in casinos with money they have recently won.

The key assumption in Barberis, Huang and Santos (2001) is that prospect theory agent's loss aversion changes over time depending on their previous investment results. If they have recently made a lot of money in the stock market, they may be less nervous, or less loss averse, because any loss they incur will be cushioned by their prior gains. However, if they have recently been burnt by painful losses in the stock market, they may be more nervous about any additional setbacks, in other words, more loss averse.

Barberis, Huang and Santos (2001) confirm Benartzi and Thaler's finding (derived in a single-period setup) in a dynamic model. They show that their model predicts large equity premia, in line with those observed in the data. The reason is that the investors in our model are loss averse: they are much more sensitive to losses than to gains, and therefore they are uncomfortable with the frequent fluctuations of the stock market and demand a large average premium to compensate them for this risk.

To understand how Barberis, Huang and Santos (2001) resolve the volatility puzzle, suppose that the stock market receives some good news about earnings. This will push the stock market up, generating substantial gains for investors. Now that they have gains, investors will be less loss averse, because these gains will cushion any subsequent losses. Since they are less risk averse than before, they are prepared to pay even more for stocks, and push stock market prices even higher. Therefore, a changing degree of loss aversion may explain why prices appear to move more than is justified by news about earnings.

The resolution of the predictability of price-earnings ratio is similar. After a good piece of earnings news, the stock market goes up, generating gains for investors, who

become less loss averse and push the stock market even further up. Since their prior gains make them feel more comfortable, investors demand a lower average return as compensation for staying in the stock market. Therefore, high prices are on average followed by lower returns, in line with the findings of predictability in the data.

2.6 Cross-sectional Return Predictability

Prospect theory has also been used to explain two most important cross-sectional stock return patterns: the predictability of book-to-market ratio (i.e., the profitability of value strategy) and the predictability of past returns (i.e., the profitability momentum strategy).

Barberis, Huang and Santos (2001) applies to the puzzles for the aggregate market. Barberis and Huang (2001) extend Barberis, Huang and Santos (2001) to multiple assets. They find that prospect theory combined with the concept of individual mental accounting works the best in explaining the cross-sectional expected return patterns, such as the value premium. The intuition is the same as that for the predictability of price-earnings ratio for the aggregate market.

Grinblatt and Han (2004) show that prospect theory and mental accounting can explain the profitability momentum strategy, or the persistence in the returns of stocks over horizons between three months and one year documented first by Jegadeesh and Titman (1993). They first show that demand for a stock by a prospect theory agents deviate from that of a fully rational investor, with the distortions being inversely related to the unrealized profit they have experienced on the stock. A stock that has been privy to prior good news has excess selling pressure relative to a stock that has been privy to adverse information. Such demand perturbation tends to generate price underreaction to public information. This distorts equilibrium prices relative to those predicted by standard utility theory. In equilibrium, past winners tend to be undervalued and past losers tend to be overvalued.

As the above mispricing gets corrected, return predictability arises: past winners, which tend to be undervalued, will continue to go up; past losers, which tend to be overvalued, will continue to go down. This leads to momentum. Interestingly, Grinblatt and Han (2004) find that the dynamic updating of the aggregate reference

prices, which occurs as shares change hands through the trading process, naturally brings a convergence of the market prices and the fundamental values of the stocks.

2.7 IPO Pricing Puzzles

There is a large literature on the pricing of initial public offerings. Two puzzles on IPOs stand out: one is the initial underpricing of IPOs; the other is the poor abnormal long-run performance of IPOs. Prospect theory has been applied to explain both.

2.7.1 IPO Underpricing

During 1990-1998, companies going public in the U.S. left over $27 billion of money on the table, where the money left on the table is defined as the first day price gain multiplied by the number of shares sold. This number is approximately twice as large as the fees paid to investment bankers, and represents a substantial indirect cost to the issuing firm. If the shares had been sold at the closing market price rather than the offer price, the proceeds of the offering would have been higher by an amount equal to the money left on the table. The investors' profits come out of the pocket of the issuing company and its pre-issue shareholders.

Many explanations have been advanced for why IPOs are underpriced (for a review, see, e.g., Ritter and Welch, 2002). Loughran and Ritter (2002) offer a new explanation based on prospect theory. Prospect theory assumes that issuers care about the change in their wealth, rather than the level of wealth. The other key aspect of the prospect theory used in Loughran and Ritter's explanation for IPO underpricing is loss-aversion (of the issuers). The issuers anchor their reference point at the IPO filling date.

Loughran and Ritter's model focuses on the covariance of the money left on the table and issuers' unanticipated wealth changes. Issuers do not get upset about the severe underpricing, especially when the issue takes place at a price above the filling range, because of their loss-averse preferences: they have gained a lot on their shares, and the underpricing is a relatively small "losses," so they irrationally aggregate the two and are still relatively happy. Issuers treat the opportunity cost of leaving money on the table as less important than direct fees. They sum the wealth loss from leaving

money on the table with the larger wealth gain from a price jump, producing a net increase in wealth for pre-issue shareholders.

Loughran and Ritter's prospect theory explanation for IPO underpricing can be recast in terms of a bargaining model in which underwriters want a lower offer price and issuing firms desire a higher offer price. When unexpectedly strong demand becomes apparent during the pre-selling period, issuing firms acquiesce in leaving more money on the table. When demand is unexpectedly weak, issuing firms negotiate more aggressively, leaving little money on the table.

Empirically, Loughran and Ritter (2002) show that most of the money left on the table comes from a minority of IPOs, i.e., those where the offer price is revised upwards from what had been anticipated at the time of distributing the preliminary prospectus. The offer price is increased in response to indications of strong demand, but it could have been increased even further. Thus, at the same time that underpricing is diluting the pre-issue shareholders of these firms, these shareholders are receiving the good news that their wealth is much higher than they had anticipated.

Loughran and Ritter's argument can be illustrated with the example of Netscapes' IPO. James Clark, a company co-founder, held 9.34 million shares. Approximately one month before going public, Netscape filed a preliminary prospectus with the Securities and Exchange Commission. This prospectus contained a projected number of shares to be issued and an anticipated price range for the offering. Based upon the midpoint of the file price range of \$12-14, the expected value of his Netscape holdings equaled \$121 million at the time that the preliminary prospectus was filed. At the closing market price on the first day of trading, his shares were worth \$544 million, a 350% increase in this component of his pre-tax wealth in the course of a few weeks. So at the same time that he discovered that he had been diluted more than necessary due to the large amount of money left on the table, he discovered that his wealth had increased by hundreds of millions of dollars. Most people would not be upset if they found themselves in this situation.

Based on Loughran and Ritter (2002), Ljungqvist and Wilhelm (2004) derive a behavioral measure of the IPO decision-maker's satisfaction with the underwriter's performance. They find that prospect theory has explanatory power for IPO market behavior. For example, IPO firms are less likely to switch underwriters for their first

seasoned equity offering when their behavioral measure indicates they were satisfied with the IPO underwriter's performance.

The prospect theory explanation also predicts a positive correlation between the filling price revision (equal to the difference between the midpoint of the filling price range and the offer price) and the first-day return. Following upward revisions, issuers are willing to tolerate higher levels of underpricing than following downward revisions. This explains the findings of Hanley (1993) that IPOs where the offer price is revised upwards see much higher first-day price jumps, on average, than those where the offer price is revised downwards.

Another unique implication of the prospect theory explanation is that market return between the filling date and the issue date is related to the IPO underpricing: much more money is left on the table following recent market rises than after market falls. Thus, the prospect theory explanation of IPO underpricing also leads to a theory of hot issue markets.

2.7.2 Long-run Under-performance of IPO

Ritter (1991) reports that the average holding period return for a sample of 1526 U.S. IPOs between 1975 and 1984 underperformed control firms of similar size and industry by nearly 29% after three years. Loughran and Ritter (1995) reported that from 1970 to 1990 the companies going public produced an average return of just 5% for the next five years, whereas a control group of nonissuing firms produced an average return of 12%. The long-run underperformance of the IPOs seems to be a consensus of the researchers, and is not restricted to U.S. markets (see Ritter and Welch, 2002, for review and discussions). Controlling for risk using the CAPM or Fama and French's three-factor model cannot eliminate the underperformance either.

Several studies argue theoretically and/or empirically that the cumulative prospect theory can explain the long-run underperformance of IPOs (see, e.g., Brav, and Heaton, 1996, Ma and Shen, 2003, and Barberis and Huang, 2004). For example, Barberis and Huang (2004) show that probability weighting can have unusual pricing effects in an economy with a positively skewed security. The key observation is that if the new security is sufficiently skewed, some investors may choose to hold large undiversified positions in it, thereby making the distribution of their overall wealth

lottery-like. Since a cumulative prospect theory investor overweights the tails of a probability distribution, he loves lottery-like wealth distributions, and is therefore willing to pay a very high price for the skewed security. Thus, the skewed security can become overpriced, relative to the price that would be set by investors who do not weight probabilities, and thus earn a very low average return.

Under cumulative prospect theory, the relationship between a securitys skewness and its expected return is nonlinear: a highly skewed security can be overpriced and earn a low average return, but a security that is merely moderately skewed is priced fairly. The security needs to be sufficiently positively skewed before prospect theory investors are willing to take undiversified position in it, since there is a trade-off between diversification and skewness preference. A sufficiently positively skewed security can be overpriced even if there are many skewed securities in the economy, again because investors may prefer a large undiversified position in just one skewed security to a diversified portfolio.

IPOs have positively skewed return distribution, probably because, being young firms, a large fraction of their value is in the form of growth options. Barberis and Huang (2004) show that based on the historical data, IPOs have sufficient skewness so that investors with cumulative prospect theory preferences calibrated to experimental data would require an average return that is several percentage points below the market return. Under cumulative prospect theory, then, the historical performance of IPOs may not be so puzzling.

Similar idea can be applied to shed light on so-called "equity stub anomalies whereby parent companies appear undervalued relative to their publicly traded subsidiaries (Mitchell and Stafford, 2002, Lamont and Thaler, 2003). If a subsidiary is valued mainly for its growth options, its returns may be positively skewed, leading investors to overprice it relative to its parent, and thereby generating a low stub value. Thus, in the presence of cumulative prospect theory investors, a firm can create value by spinning off subsidiaries that are valued mainly for their growth options. If a subsidiary of this kind is traded as a separate entity, its stock is likely to have a positively skewed return distribution. If investors overprice such securities, the firm will be more valuable when its parts are traded separately, than when they are traded as a single bundle.

3 Conclusion

Prospect theory is a descriptive model of decision making under uncertainty. Under prospect theory, people evaluate risk using a value function that is defined over gains and losses, is concave over gains and convex over losses, and is kinked at the origin; and using transformed rather than objective probabilities by applying a weighting function.

This paper only reviews a portion of of topics that prospect theory can help us understand. Other areas where prospect theory has interesting applications includes commercial banking (e.g., Johnson, 1994, Godlewski, 2004), investment banking (e.g, Willman et al, 2002), hedge funds (e.g., Siegmann and Lucas, 2002, Kouwenberg and Ziemba, 2004) venture capital firms (Dubil and Maretno, 2003), and analyst behavior (e.g., Ding, Charoenwong, and Seetoh, 2004). This list is still growing fast. Prospect theory, together with other behavioral concepts such as mental accounting and narrowing framing, may offer a new paradigm for understanding behavior of participants in financial markets as well as market prices.

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Figure 1: **Prospect Theory Value Function**

This figure plots an example of the S-shaped prospect theory value function, generated by the following:

$$\begin{aligned} U(W) &= \frac{(W-R)^{1-\gamma}}{1-\gamma}, \qquad \text{if } W \geq R; \\ U(W) &= -\lambda \frac{(R-W)^{1-\gamma}}{1-\gamma}, \quad \text{if } W < R \end{aligned}$$

where R is a reference level, $\gamma = 0.5$ and $\lambda = 2.25$.