Beyond Cap Weight

The empirical evidence for a diversified beta

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For over 40 years, our industry has relied on the capital asset pricing model (CAPM) beta and the capitalization-weighted market portfolio for asset allocation, for market representation and for our default core equity investments. This elegant worldview is now under siege from various directions.

The "fundamentalists" advocate a portfolio that weights companies in accordance with the recent economic scale of their businesses, thereby resembling the composition of the economy rather than the composition of the stock market. The "minimum variance" crowd points to the value of consistency between investor objectives and portfolio construction. The "egalitarians" advocate equal weighting. Historically, these alternative index strategies have delivered higher return and lower CAPM beta, which can help an investor to target either more return or less risk, or a bit of both. Each of these strategies—along with the ever-dominant cap-weighted indexes—has strengths and weaknesses, some minor and some major.

The cap-weighted standard is also facing a more subtle source of attack. Increasingly, investors are reassessing their risk budgets, usually downward. This can create pressure to move from active into passive strategies and to lower a fund's exposure to an undiversified single-factor equity risk. But, can we lower our risk profile without abandoning our return goals? Perhaps it is time to consider a bigger tent, allowing for the merits of multiple broad-market indexes and multiple betas.

We explore the comparative merits of four major categories of quasi-passive "index" construction. We do so from a global perspective. And we explore the surprising efficacy of combining multiple strategies into a diversified beta portfolio.

Introduction

Historical concepts regarding market efficiency and single-factor beta are losing favor, as markets have whipsawed even the best-diversified portfolios. Just as many investors are increasing exposure to passive strategies, they face a new and unsettling prospect of "benchmark regret," to borrow from the terminology of behavioral finance, as it's no longer clear that market-capitalization weighting (cap weight) is the only legitimate benchmark or core portfolio choice. In fact, institutional investors can choose from a wide array of alternative beta strategies, including equal weight, minimum variance and economic size (also known as the fundamental index approach), to name a few.

These alternatives have generally offered better returns or lower volatility, or both, when compared with cap weight, both in historical tests and on live assets, albeit over a shorter span than cap weight and on a smaller asset base. If the performance advantage of the alternatives persists, a decision to commit to cap weight and to ignore the alternatives may someday be second-guessed as an overly narrow and costly mistake. Although some in the mainstream indexing community dismiss these alternative beta strategies as cleverly packaged active management strategies, we believe that these alternatives provide useful alternatives to the single-beta cap-weighted index portfolio.[1]

The historical record for each of these alternative index strategies suggests some particular competitive advantage. Equal weight has the longest live track record of added value, dating back to the early days of the Value Line index; minimum variance offers the highest historical Sharpe ratio and lowest risk; economic scale portfolios offer the highest information ratio; and cap weight offers vast scalability, theoretical purity—in an efficient market, the others should not win on a risk-adjusted basis—and, of course, the lowest tracking error relative to the stock market, which is inherently cap-weighted.

In the rapidly changing world of indexing, any investment decision is an active choice, even a switch from active into passive exposure. The decision to invest passively provides only a starting point for determining *which* passive or quasi-passive approach best meets an investor's needs. Cap weight is no longer the only compelling choice, not to mention that there are many cap-weighted indexes to choose from.

Our research focuses on a few of the "index" strategies that are gaining traction in the marketplace and explores the potential value of a diversified approach in our quest for beta. Some call these new ideas beta-prime, some call them enhanced indexing, still others dismiss these approaches as active management in sheep's clothing. Whatever we call them, few would deny that they are fast changing the investing landscape.

De-leveraging And Noncap Weighting

Many investors are reducing their risk budget—some term this de-risking—for a host of reasons. Some are doing so to reduce the sometimes-frightful gap between assets and liabilities. Others are acting out of a fear of an over-leveraged global economy and the impact that this leverage can have on capital markets' volatility. Others are doing so to rein in the impact that funding ratio volatility can have on their earnings. And many are doing so because of a fear that the current low yield for stocks, paired with an uncertain inflation outlook, casts a cloud of doubt over the future prospects for the much-vaunted equity risk premium.

Keith Ambachtsheer, Marty Leibowitz and Peter Bernstein have noted repeatedly over the past quarter-century that there's a peculiar link between inflation and real equity returns. When inflation is low and stable, equity valuation levels rise and returns are handsome. When deflation strikes, government bond prices soar and equity valuation levels crater. The correlation between stocks and bonds tumbles. And, when reflation leads to sustained high rates of inflation, bond *yields* soar (bond prices fall) and equity valuation levels tumble. The correlation between stocks and bonds soars.

It is far beyond the scope of this paper for us to weigh in on the sometimes vitriolic debate about the relative risk of deflation, reflation, stagflation or hyperinflation in the years ahead. However, it is well worth noting that there are remarkably few observers of current markets who harbor hopes for a benign move back to the low, stable and well-orchestrated rates of inflation of the 1990s and pre-crash 2000s. Because either extreme—deflation or stagflation—tends to be rather savage to equity valuation levels and real earnings growth prospects, these fears fuel additional calls for de-risking.

De-risking an institutional portfolio can mean many things. Typically, de-risking involves investors shifting from riskier assets into more defensive assets. This can mean lower overall equity allocations, lower beta strategies and/

or allocating more of our risk budget to passive management and away from active management. Shifting assets from active to passive management is a popular choice for many reasons, including lower total costs and the empirical evidence that most—but assuredly not all!—active managers fail to add value.

A more subtle reason for de-risking, which can be observed but not measured, is public anger at a perception of "Wall Street greed." The low costs of cap weight, *and its new quasi-index brethren*, mean that more of the return of the holdings flows to the investors, and less to the managers, brokers, custodians and other intermediaries. This pleases many customers!

For all of these investors, the noncap-weighted "index" strategies are important additions to the investment tool kit. If these strategies offer higher return and/or lower volatility on average over time, as history would suggest, at a fraction of the cost of fully active strategies, then investors can choose to reach for more return at the same portfolio risk or they can choose to reduce equity market exposure without necessarily reducing the return.

Index Alternatives

Of the alternatives, we have chosen to explore—and combine—the four approaches that are garnering the most attention as alternative core equity strategies. As we delve into their characteristics, let's also examine the principles and tacit core assumptions that lay a foundation for each.

Some of the key attributes for the four index strategies are summarized in Figure 1. For purposes of evaluating global results, across all four types of strategies, currencies were hedged by using interest-rate differentials to approximate the impact of continuous hedging; this did not include the impact of actual hedge costs (Lazard, 2009).

Cap Weight

Market capitalization remains immensely popular as the incumbent and theoretically efficient choice, despite doubts about whether its core theoretical underpinnings—the efficient market hypothesis (EMH) and CAPM—are precisely correct. Cap weight tacitly assumes that share-price-implied consensus expectations, regarding the net present value of each company's future growth prospects, are an unbiased view of the future. Furthermore, cap weight offers very low turnover, trading costs and tax consequences, which the newer alternatives can't quite match.

As EMH and CAPM gained traction in academia, the theoretical result—that a single portfolio could be optimal—was revolutionary. The theoretical purity of cap weight, along with the difficulties faced by the *average* active manager, in time gave rise to passive investing. The growth in "index funds" was fueled by the historical fact that the average active manager has had a hard time beating cap-weighted indexes, after taking account of fees and transaction costs.

No student of the capital markets should find this the least bit surprising. After all, if we divide the market into the passive, cap-weighted indexes and the combined holdings of all active, noncap-weighted portfolios—including individual investors—the former matches the market in both holdings and performance, which means that the latter must also match the market, before costs. So, net of costs, the noncap-weighted active managers must collectively lag cap weight. None of this requires market efficiency, nor is it necessary to believe that we cannot "beat the market." We need only admit that winning active managers must have losing managers on the other side of their trades! Even as indexing gained traction, a growing body of empirical evidence suggested that patient investors could achieve above-market performance, with statistical significance, most notably with a value tilt.

If EMH and CAPM are mere approximations of the real world, then the assured dominance of cap weight, on a risk-adjusted basis, evaporates. Suppose we believe that markets are inefficient, and that investors are subject to errors that result in share prices that deviate from their fair valuations. When investors construct portfolios that weight companies proportional to capitalization, they inherently overweight the overpriced stocks and underweight the underpriced stocks. This truism has been acknowledged by many in the indexing community, and dismissed because of the equally relevant truism that we cannot know which companies are over- or undervalued. But, if we can sever the link between over- or undervaluation and portfolio weight, perhaps we can improve upon cap weight. Or so the "fundamentalists" suggest.

Figure 1

Portfolio Construction Comparison

Portfolio/Index	Relative Size Determination	Required Forecasts	Turnover and Trading Costs
Cap Weight	**Market Cap**	**None[a]**	**Minimal**
Economic Scale	Fundamental size (economic footprint)	None	Somewhat higher than market cap
Global Equal Weight	Equal	None[b]	High
Equal Weight Cap 1000	*Equal*	*None[b]*	*High*
Equal Weight Econ 1000	*Equal*	*None[b]*	*High*
Minimum Variance	Risk contribution (equal marginal change in risk)	Volatility and correlations	Somewhat higher than market cap

[a]The weight tacitly reflects a market consensus forecast for future risk-adjusted shareholder distributions.
[b]There is a selection bias issue explored in the text: Which stocks do we select for our equal weighting?
Sources: Research Affiliates, LLC, and Lazard Asset Management

Economic Scale (Or Fundamental Index)

The economic scale approach uses a company's fundamental economic size—weighting companies according to sales, cash flow, book value and dividends, then averaging the four measures—both to select the 1,000 largest companies and then to assign portfolio weights to each company in an index.[2]

The "fundamentalists" point out that *if* the market is inefficient and prices may stray above or below a company's future true fair value (which Bill Sharpe whimsically terms its "clairvoyant value," because only a clairvoyant could know that value), the cap weight of every overvalued company will be above its fair value weight, and the cap weight of every undervalued company will be below its fair value weight. This truism means that a cap weight portfolio will experience a performance drag relative to a clairvoyant-value-weighted portfolio.[3] This is not controversial. As we cannot know future cash flows on our investments, and so cannot construct the clairvoyant-value-weighted portfolio, so what?

Suppose we break the link between over- and under-weighting relative to clairvoyant value weight (the clairvoyant error in weight) and over- and undervaluation relative to clairvoyant value (the clairvoyant error in price). In other words, suppose that there's no correlation between the two "error gaps"—clairvoyant error in price and clairvoyant error in the weight in our portfolio. We still have overvalued and undervalued companies; we still have companies that are above or below fair value weight. But, these are no longer identically the same lists. The errors cancel.

If we weight companies by their fundamental economic size, we enjoy many of the attractive attributes of the cap weight portfolio, such as liquidity, low turnover, scalability and objectivity. But we no longer assuredly overweight the overvalued stocks and underweight the undervalued stocks, relative to the unknowable "clairvoyant value weight" of each company. In so doing, *if the market is inefficient and the price contains a mean-reverting error*, we arguably eliminate the greatest Achilles' heel of cap weight: the performance drag associated with its assured overreliance on the overvalued companies.

Selecting and weighting companies for a stock market index, using fundamental economic measures of company size, was introduced by Arnott, Hsu and Moore in 2005. Some such approaches rely on single measures, such as dividends or revenues. Others rely on multiple measures. The result is a portfolio where position size is proportional to some measure of a company's "economic footprint." Just as our footprint in the sand at the beach has multiple measures—length, width, depth—the footprint of a company in the economy has multiple measures. The FTSE RAFI methodology, which we use in this research, relies on four measures of the size and recent success of a company, including sales, cash flow, book value and dividends. This methodology creates a representative portfolio, weighted to mirror the look and composition of the publicly traded economy, rather than the look and composition of the stock market.

The "fundamentalists" argue that this economic scale serves as an anchor for contratrading against the constantly shifting expectations of the market, and that this contratrading is the primary profit mechanism of economic scale portfolios. Some even suggest that economic scale strategies do not earn an alpha at all; rather, they suggest that cap weight incurs a negative alpha, *against its opportunity set*, which economic scale partly corrects.

The economic scale portfolios do not have a monopoly on this advantage: The same holds true for any index method that provides a steady anchor for contratrading against the market's most extreme bets. This same argument may be made for any weighting scheme that does not take share price into account when setting portfolio weights, which brings us to the other two index structures that we wish to explore in this paper.

Equal Weight

The equal weight approach assigns an equal weight to each company in an index, thereby tacitly assigning zero information value to all public and private information about a company except for its inclusion in the source index. For instance, the S&P 500 Equal Weight Index (S&P EWI) tacitly assigns value to a stock's inclusion or exclusion from the S&P 500 Index, but no value to any differentiating information, which might lead us to prefer any one company over any other. Equal weighting was the basis for the first index futures (the Kansas City Value Line Index futures from the early 1980s), has the longest history of the "index" alternatives, and provides an interesting counterpoint to cap weight.

Suppose we assume that it is impossible for any investor to predict a security's risk or return, or the covariance matrix. Then, it follows that holding an equal amount of each investable security results in the portfolio with the lowest predicted risk, at no sacrifice to our expected return. Put another way, if the cap weight portfolio reflects the view that the aggregate investor universe fully incorporates risk and return forecasts, then equal weight assumes that the aggregate investor universe has zero ability to forecast *anything*.

For practitioners, the elegant simplicity of an equally weighted portfolio is compromised by implementation issues. Because equal weight means that we hold small companies on the same scale as large ones, the strategy results in higher transaction costs and lower capacity than cap weight. Still, absent trading costs and any view on forecasting return or risk, equal weighting has considerable appeal on a risk-return basis.

One nuance that has received startlingly little attention in the academic and practitioner journals is: *Equal weighting of what index?* If cap weight has a bias toward including overvalued companies, then equal weight may exacerbate this bias. For instance, a clairvoyant might assert that the future prospects of 150 companies in the S&P 500 do not justify inclusion in the index. Their "clairvoyant value" market cap is too low. Because they will assuredly under-perform eventually, they will pull down the S&P 500 return relative to our mythical clairvoyant value portfolio. But, where these stocks might comprise 5-10 percent of the S&P 500, they comprise 30 percent of the S&P EWI. We have a possibly severe "selection bias" problem!

So, what list should we use?

Suppose, instead, we equal-weight the 1,000 largest companies, measured by their economic footprint. We know that the 1,000 largest market-cap stocks have considerable overlap with the 1,000 largest companies measured in terms of economic footprint (a Fundamental Index® portfolio). Large companies are usually large-cap, and vice versa. There would typically be 700-800 overlapping companies. So, if we equal-weight a large-cap 1,000-stock index, there will be 200-300 companies in the portfolio that are—by definition—small companies trading at lofty enough multiples that they become large-cap. Empirically, many of these subsequently disappoint.

Reciprocally, by equal-weighting the 1,000 largest companies selected based on fundamental economic scale, we include 200-300 companies that are—by definition—large companies trading at deep enough multiples to be small-cap. While these might comprise 3-5 percent of the economic scale portfolio, they comprise 20-30 percent of the equally weighted economic scale portfolio. Many of these are of good clairvoyant value and many are not; it's a more random result. The empirical result is sharply higher performance than equally weighting an index that has been selected by market cap.

We should note that no one has built a product based on equally weighting a Fundamental Index portfolio, but we think it's a very interesting idea. It has similar merits and demerits, when compared with the now widely accepted equal-weight portfolios based on cap-weighted indexes such as the S&P 500. In an efficient market, these two equal-weight portfolios should have much the same return. In practice, they do not. Accordingly, to mitigate the potentially serious problem of selection bias, we construct two equally weighted 1,000-stock portfolios—cap weight and economic scale—and then equal-weight the 1,000 largest, based on the combined rankings of size. And we test all three. Applied globally, we get our global equal weight portfolio.

Minimum Variance

Minimum variance portfolios are designed to reduce portfolio risk. In an efficient market, this should not improve our risk-adjusted returns. But, if equity returns are not linearly related to beta, as CAPM predicts, it may generate high risk-adjusted returns. This approach, introduced in the early 1990s, has been gaining traction recently. It builds portfolios without reference to a benchmark, by using historical measures of risk with the goal of minimizing the portfolio volatility. Its efficacy depends on the market mispricing risk. In a world increasingly focused on risk, it is unsurprising that this concept is gaining attention.

Investors have traditionally created equity portfolios that manage risk relative to market indices; less attention has been paid to the question of which index best meets investors' needs. Minimum variance portfolios are constructed to create high risk-adjusted returns by minimizing volatility without reference to return expectations.[4] Haugen and Baker (1991) were pioneers in this domain; their U.S.-focused research principally concluded that, due to investor restrictions on short selling, tax situations, and risk and return expectations, portfolios could be constructed that dominated the market portfolio in terms of risk-adjusted returns.

Alternatively, this incremental return could potentially be explained by the presence of additional unidentified risk sources in the low-volatility portfolio. An alternative explanation of the incremental return is that these portfolios systematically favor risks that the market has mispriced. Risk can be mispriced due to differences in measurement as well as the relative importance that investors place on different measures of risk. The classic definition of risk as the volatility of total return is inconsistent with investor experience. Falkenstein (2009) suggests a utility function that measures risk within the context of relative wealth and that this is an outcome of investor preference for status. This perspective is consistent with the institutional investor focus on information ratio as the preferred measure of risk-adjusted returns. Evidence that risk preferences vary among individual investors is provided by Dorn and Huberman (2009), who examined a large number of broker accounts and found that holdings tended to cluster by volatility. Portfolio risk considerations are secondary to return expectations and the comfort of stocks that are within preferred risk habitats.

A related opportunity has been identified to invest in stocks with low volatility. This portion of the universe has been found to have greater-than-market returns, while stocks with high volatility empirically tend to deliver lower returns. Ang et al. (2006) documented this effect while researching a broad universe of U.S. stocks and concluded that the effect could not be explained by size, book-to-market, momentum and liquidity. Similar effects were found in European and Japanese markets by Blitz and van Vliet (2007), who controlled for illiquidity and found the results to still be intact: Low volatility stocks deliver higher risk-adjusted returns, even when controlling for value and size. Investors have used this approach within both active and quasi-passive styles.

Minimum variance offers an interesting challenge for our purposes: There are as many ways to construct a minimum variance portfolio as there are ways to measure past, present or future risk! The strength of the academic evidence in favor of minimum variance has prompted benchmark providers, such as MSCI Barra, to calculate their own minimum variance portfolios. The MSCI World Minimum Volatility Index, launched in 2008, experienced approximately 30 percent lower volatility than the MSCI World Index over the simulated history (1995-2007), with a 50 percent improvement in the Sharpe ratio. But, the MSCI Barra methodology is proprietary, so we cannot replicate it for our purposes.

It is not our intent to be exhaustive in exploring the many permutations of minimum variance, so we have adopted the approach that our authors from Lazard use in one of their minimum equity risk strategies. Risk measurement is based on a diversified approach that incorporates multiple measures, including interest rates, oil prices, region and sector as well as size, yield and growth, as calculated by the Northfield global risk model. We minimize the *absolute* risk of the portfolio, subject to some constraints to assure broad diversification and investability.[5]

Combining The Indexes

These methods provide discrete choices to the investor, with very different and surprisingly complementary characteristics. This is by no means an exhaustive list. For instance, two organizations in France, TOBAM and EDHEC, have developed very interesting "maximum diversification"[6] and "efficient index" portfolios. The TOBAM team, formerly members of the Lehman Brothers quantitative research group, constructs a "maximum diversification" portfolio that has an equal and lowest-possible correlation with its constituent holdings, and for which all excluded assets would boost the correlations, if included. The EDHEC "efficient index" portfolio is based on presuming that return is linearly linked to a general measure of total risk (semi-deviation) and then using Markowitz mean-variance optimization to identify the tangency portfolio. Both ideas are fascinating variants of the broad concept of minimum variance. For another example, see Held (2008), for equal-weighted sector rebalancing.

We think the four strategies that we include in our research are the most widely accepted passive and quasi-passive alternatives. They can be combined to create a compelling investment—*and* a "diversified beta"—that incorporates many of the historical advantages of passive portfolios while perhaps earning higher returns or experiencing less risk. This paper reviews the comparative characteristics and returns for each approach. We then consider the use of a diversified set of methods in combination and provide some concluding comments and suggestions for further research.

To test whether investors are better off using combinations of passive strategies, we look at two additional strategies. The first is an "efficient beta," which is calculated by equally weighting cap weight, economic scale and minimum variance strategies. We call this combination "efficient beta," as it has relatively low transaction costs and substantial investment capacity. The second equally weights all four indexes. We tacitly assume monthly rebalancing of the three or four strategies back to equal weights. Trading costs are also explored, even though the resulting turnover for all of these strategies is relatively slight, so this layer of costs will be minimal.

Currencies were hedged by using interest-rate differentials to approximate the impact of continuous hedging and did not include the impact of actual hedge costs. As is reasonably standard in published index returns, our results do not reflect transaction costs. However, trading costs matter, even with low-turnover index and quasi-index strategies. The impact of transaction costs can be inferred based upon the annual turnover and average market cap of the respective portfolios, as we briefly summarize later.

Relative Performance

Our research covers global equity strategies that are fully hedged back into U.S. dollars, covering the period January 1993 through June 2009. This relatively recent span is a function of available global data. We required sufficiently detailed information across global markets to permit construction of *all four strategies*, and information complete enough to include all nonsurviving companies of sufficient scale to enter any one of our portfolios.

Critics may point to the short history in this study, style biases in these alternative core portfolios, implementation challenges and so forth. Because we wanted to test these

Figure 2

Annual Comparative Returns, 1/1993-6/2009

Year	Cap Weight	Economic Scale	Equal Weight	*Equal Econ Weight*	*Equal Cap Weight*	Minimum Variance	Efficient Beta	All Four Combined
1993	18.8%	26.5%	21.3%	*26.4%*	*19.5%*	20.3%	21.9%	21.8%
1994	0.3%	1.6%	0.7%	*1.4%*	*-0.7%*	-2.3%	-0.1%	0.1%
1995	20.2%	22.7%	20.3%	*20.4%*	*20.5%*	22.6%	21.8%	21.5%
1996	16.6%	19.0%	16.0%	*17.8%*	*15.7%*	20.7%	18.7%	18.1%
1997	22.5%	25.0%	19.7%	*19.5%*	*20.3%*	28.6%	25.4%	24.0%
1998	19.9%	17.0%	16.0%	*15.2%*	*16.2%*	15.1%	17.4%	17.1%
1999	32.7%	27.7%	20.7%	*20.3%*	*24.3%*	8.8%	22.8%	22.3%
2000	-8.3%	8.8%	9.6%	*13.2%*	*-1.5%*	14.0%	4.6%	5.9%
2001	-14.0%	-4.7%	-4.7%	*-0.4%*	*-9.3%*	-1.2%	-6.7%	-6.2%
2002	-22.3%	-20.0%	-18.7%	*-16.4%*	*-20.3%*	-13.5%	-18.6%	-18.6%
2003	26.5%	30.5%	31.5%	*35.1%*	*29.3%*	17.6%	24.8%	26.4%
2004	12.6%	13.8%	17.3%	*18.4%*	*16.4%*	20.6%	15.6%	16.0%
2005	18.6%	19.5%	22.3%	*23.3%*	*22.4%*	20.3%	19.5%	20.2%
2006	18.3%	22.2%	20.8%	*21.8%*	*19.7%*	22.3%	20.9%	20.9%
2007	7.4%	6.2%	4.8%	*2.9%*	*6.5%*	6.3%	6.6%	6.2%
2008	-38.6%	-38.4%	-40.1%	*-39.9%*	*-41.1%*	-29.1%	-35.5%	-36.6%
2009	5.7%	7.7%	10.3%	*14.3%*	*10.2%*	0.5%	4.7%	6.1%
Hi/Lo	3 vs 8	3 vs 0	—	*4 vs 0*	*0 vs 2*	7 vs 5	—	—
Win/Loss	—	14 vs 3	11 vs 6	*12 vs 5*	*10 vs 7*	11 vs 6	11 vs 6	11 vs 6

Sources: Research Affiliates, LLC, and Lazard Asset Management

Figure 3

Return Characteristics, 1/1993-6/2009

Portfolio/Index	Ending Value of $1	Geometric Return	Volatility	Sharpe Ratio	Excess Return vs. Cap-Weighted	Tracking Error vs. Cap-Weighted	Information Ratio	*t*-Statistic for Excess Return
Cap Weight	**2.78**	**6.39%**	**14.50%**	**0.18**	—	—	—	—
Economic Scale	4.44	9.46%	14.35%	0.40	3.07%	5.00%	0.62	2.31*
Global Equal Weight	3.83	8.48%	14.44%	0.33	2.10%	4.65%	0.45	1.71
Equal Weight Cap 1000	*3.12*	*7.14%*	*15.10%*	*0.22*	*0.75%*	*3.09%*	*0.24*	*1.05*
Equal Weight Econ 1000	*4.82*	*10.00%*	*14.86%*	*0.42*	*3.61%*	*5.98%*	*0.60*	*2.31**
Minimum Variance	4.31	9.26%	10.70%	0.52	2.87%	7.39%	0.39	1.20
Efficient Beta	3.80	8.43%	12.81%	0.37	2.04%	3.71%	0.55	1.83
All-Four-Combined	3.81	8.45%	13.18%	0.36	2.06%	3.75%	0.55	1.89

* Significant at 95% confidence level.
Sources: Research Affiliates, LLC, and Lazard Asset Management

ideas on a *global* scale, and across all four methods, the research is necessarily rooted in a relatively short historical span, covering just over 16 years of equity market results. However, the results mirror the country-by-country results of others, testing single methods, and mirror the longer-span results observed in less-than-global applications. Although this history is not as long as we might prefer, these results do span several market cycles, including much of the secular bull market of the 1990s and the secular bear market since.

For a proper apples-to-apples comparison, we created a ***simulated*** cap weight portfolio. Because the economic scale and global equal weight portfolios each span 1,000 companies, we wanted to create a cap weight developed markets index that is as objective as possible. This portfolio comprises the 1,000 largest companies domiciled in the 23 developed economies contained in the FTSE and MSCI developed world indexes, selected and then weighted by market cap. The methodology is analogous to a developed world "Russell 1000." It will come as no surprise that it tracks very closely with the published currency-hedged FTSE and MSCI developed world indexes.

The year-by-year results, in Figure 2, show that there are markets in which each will shine. Cap weight is best of the bunch in 1998, 1999 and 2007, years in which growth won handily and active managers generally struggled. Economic scale was best in three years and the *equal-weighted* economic scale portfolio was best in another four years. And minimum variance was best in seven years, surprisingly not just in the weak years for stocks, when shunning risk would be expected to win.

Of course, the composite strategies can never be best or worst, because that would require them to beat all of their constituent portfolios or lose to them all, over an individual year. This affects the combined global equal weight portfolio, as well as the efficient beta and all-four-combined strategies.

They also exhibited differing "batting averages" when compared with cap weight, ranging from 14 wins and 3 losses for economic scale, to 10 wins and 7 losses for equal-weighting the cap weight portfolio. We should point out that, with only 17 years (actually 16½!) of data, a batting average of 13-to-4 is required for 95 percent two-tail confidence statistical significance.[7] Although many practitioners think of 17 years as a long time, statistically it is not.

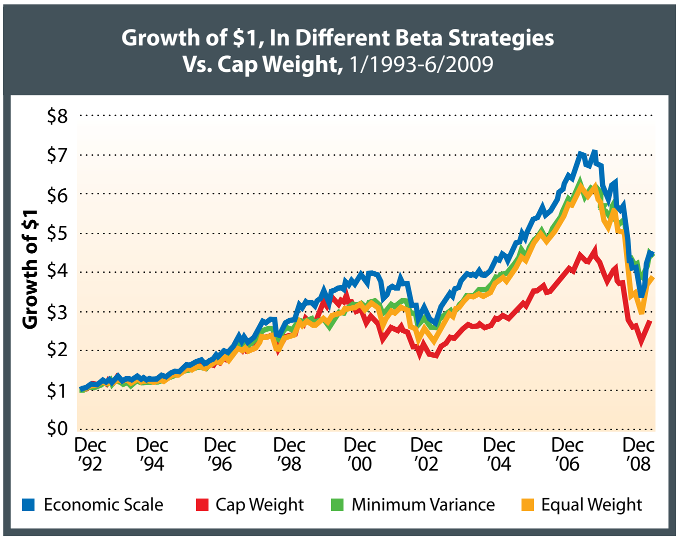
As Figure 3 shows, all of the noncap-weighted strategies offer superior performance over the simulated cap weight strategy over this span. Interestingly, the two subindexes of the global equal weight portfolio have very different results. When we equal-weight the 1,000 largest companies by market capitalization, we outpace the market cap portfolio by some 75 bps per annum with notably higher risk. When we equal-weight the 1,000 largest companies, based on a blend of four measures of the scale of a company's business, we outpace the economic scale portfolio by a smaller margin of 54 bps, again with higher risk. When we combine the two universes (equal-weighting the 1,000 largest, based on the sum of the two rankings), we beat cap weight by 210 bps per annum while falling less than 100 bps behind the economic scale portfolio, with risk very near the average of cap weight and economic scale.

Each strategy has its own strengths and weaknesses. Cap weight tautologically has the lowest tracking error and *should* maximize risk-adjusted return—if EMH and CAPM hold fully and perfectly true. Minimum variance achieves its objective with the lowest volatility of 10.70 percent and highest Sharpe ratio of 0.52. The economic scale portfolio, measured relative to the cap weight portfolio, has the highest information ratio, 0.62, and ties with its own equal-weight variant for best statistical significance for alpha, with a *t*-statistic of 2.31. Equal-weighting the cap weight portfolio offered the lowest tracking error of the noncap-weighted strategies, but also delivered the highest volatility.

The combinations are surprisingly robust. When investors are uneasy about a singular reliance on cap weight for their core holdings—and, so, choose either beta combination strategy—both beta combinations result in higher performance and lower volatility when compared with an exclusive use of the cap weight strategy. Even relative to the constituent noncap-weighted strategies, for *both* of the combinations that we test herein, we wind up with an array of attractive attributes:

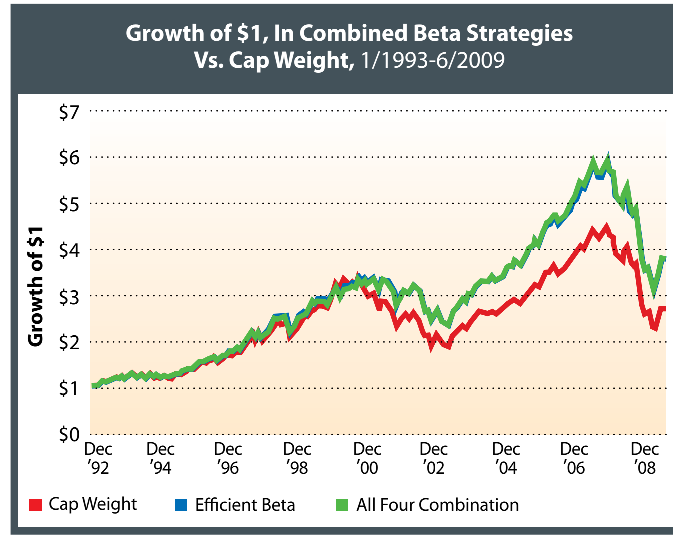
• Our return is modestly higher than the average of the individual strategy constructs, while our risk is similar, leading to a slightly better Sharpe ratio than the average of the constituent single portfolios.

Figure 4



Sources: Research Affiliates, LLC, and Lazard Asset Management

Figure 5



Sources: Research Affiliates, LLC, and Lazard Asset Management

• The tracking error is about 10 percent less than the average of the constituents, leading to an information ratio that's quite a bit higher than the average.

The same holds true for the combination of all four index methods (cap weight, economic scale, global equal weight and minimum variance). Indeed, adding global equal weight to our efficient beta leads to results that are almost indistinguishable on most dimensions from our efficient beta portfolio.

The main conclusion that we draw from these results is that all five alternatives to the cap weight portfolio, as well as both combined strategies, have historically dominated cap weight in returns and/or risk-adjusted returns. A classical return attribution would suggest that this is at least partly due to the size and value tilts inherent in these various strategies. Alternatively, as we've suggested in other papers, this advantage is perhaps because the noncap and combined strategies all contratrade against the market's constantly changing expectations, as reflected in a company's share price and market capitalization.

Intuitively for an investor, these results are best demonstrated with conventional dollar growth charts. The behavior of all of these portfolios is very similar. Naturally, bull and bear markets and the corresponding peaks and troughs happen at roughly the same time. One notable exception is the market peak in 2000. The noncap-weighted portfolios all peaked over a year later than the cap weight portfolio. By not loading up on the Ciscos and Nokias of the world, we are less hurt by the collapse of the tech bubble.

Consistent with the strategy's design, Figure 4 shows the lower risk of the minimum variance portfolio, as is clearly evident in the more stable return stream. When compared with a market capitalization index, the returns are weaker in extreme market rallies and more resilient in bear markets. It is a nicer ride—to very nearly the best end-point wealth!—with stable returns for investors who are concerned about total volatility. We should readily acknowledge, however, that the tech bubble of 1999-2000 would have tried the patience of any adherent to the noncap-weighted alternatives!

For the conservative investor who does not like putting all eggs into the same "beta basket," a good alternative may be to diversify among different beta strategies. The efficient beta portfolio and all-four-combined portfolio preserve some of the good characteristics of the economic weighting and minimum variance approaches. The cumulative performance of the blended strategies is displayed in Figure 5. It is remarkable to note that the two combined strategies—despite the all-four-combined including a very different global equal weight strategy—are near-identical in returns, risk and other characteristics. Adding global equal weight does nothing for us.

When adjusting results for beta, relative to our simulated cap weight portfolio, economic scale and minimum variance have positive CAPM alphas of 3.53 percent and 5.17 percent, respectively, as shown in Figure 6. Betas of the strategies are 0.93 and 0.94 for economic scale and global equal weight, respectively; for minimum variance, the beta is vastly lower, at 0.64, exactly as we should expect for a low-volatility strategy that is not constructed with reference to a benchmark.

The striking difference that comes from our risk-adjusted results is that the statistical significance soars. If much of our tracking error is attributable to a lower beta, then the residual risk is actually smaller than the tracking error, notably for minimum variance; for much the same reason, the added value is larger than it seems. Adjusted for market risk, only the global equal weight portfolio and the equally weighted portfolio, which is drawn from cap weight, lack statistical significance. The economic scale and minimum variance portfolios loft to startling statistical significance for so short a span. *And, in a gratifying surprise, the efficient beta portfolio rivals the highest risk-adjusted information ratio of any of its constituent strategies, with higher statistical significance than any single strategy.*

In Figure 7, we examine the outlier risks of the various portfolios. All portfolios have excess kurtosis and negative skewness, which are well-known characteristics of most equity investing strategies. Drawdown characteristics are similar, with minimum variance showing the smallest drawdowns and global equal weight showing the largest. Reciprocally, minimum variance sharply reduces the largest gains in strong months and quarters, though this weakness disappears over 12-month spans. The minimum variance portfolio also has the highest negative skewness. Of course, this asymmetric characteristic in the minimum variance return distribution—more extreme losers than winners—is mitigated by the significantly lower volatility of this strategy as noted in Figures 2 and 3.

The combined efficient beta portfolio's outlier characteristics are more similar to those of the cap weight portfolio than *any* of the individual noncap strategies. This may serve to reassure the risk-averse investor: Moving from a singular reliance on cap weighting to a more diversified approach does not subject our portfolio to any significant increase in the downside risk. In each time span, efficient beta's greatest win is larger than for cap weight, while its greatest loss is smaller.

Figure 8 reconstructs the Fama-French-Carhart methodology, based on the global cap-weighted 1,000 stock portfolio.[8] In this multivariate context, the beta relative to the cap weight soars well above the betas shown in Figure 6. The SMB size factor loading is far smaller than most might expect, because other risk factors—notably beta—can proxy in a multivariate regression. Meanwhile, economic scale and minimum variance portfolios both have quite a substantial HML value tilt relative to cap weight, while the equally weighted portfolios have only modestly more HML value tilt than the strategies from which they were sourced. Because global equal weight is partly sourced from economic scale, which has a large HML value tilt, it also has a reasonably large value tilt.

Even though much of the alpha is driven by size and value effects, both economic scale and minimum variance have quite sizable annualized alphas, of 2.07 percent and 1.81 percent, respectively, *net of the Fama-French-Carhart factors*. The alpha is considerably more significant in all cases than the simple CAPM alphas shown in Figure 6, even though the alphas are smaller once we net out the impact of size, value and momentum effects. Indeed, all of the noncap-weighted strategies, without exception, exhibit far more statistical significance net of these "style tilts" than they do on either a simple value-added or a CAPM alpha basis.

Figure 6

CAPM Characteristics, 1/1993-6/2009

Portfolio/Index	Ending Value of $1	Geometric Return	Correlation with Cap-Weighted	CAPM Beta vs. Cap-Weighted	Excess Return vs. Cap-Weighted	CAPM Alpha vs. Cap-Weighted	Information Ratio of Alpha	*t*-Statistic for CAPM Alpha
Cap Weight	**2.78**	**6.39%**	—	—	—	—	—	—
Economic Scale	4.44	9.46%	0.94	0.93	3.07%	3.53%	0.72	2.56*
Global Equal Weight	3.83	8.48%	0.95	0.94	2.10%	2.45%	0.54	1.91
Equal Weight Cap 1000	*3.12*	*7.14%*	*0.98*	*1.02*	*0.75%*	*0.62%*	*0.20*	*0.95*
Equal Weight Econ 1000	*4.82*	*10.00%*	*0.92*	*0.94*	*3.61%*	*3.99%*	*0.67*	*2.48**
Minimum Variance	4.31	9.26%	0.87	0.64	2.87%	5.17%	0.99	2.72**
Efficient Beta	3.80	8.43%	0.97	0.86	2.04%	2.96%	0.96	2.90**
All-Four-Combined	3.81	8.45%	0.97	0.88	2.06%	2.84%	0.86	2.68**

* Significant at 95% confidence level. ** Significant at 99% confidence level.
Sources: Research Affiliates, LLC, and Lazard Asset Management

Figure 7

Outlier Risks, 1/1993-6/2009

Portfolio/Index	Skewness	Excess Kurtosis	Maximum Monthly Return	Minimum Monthly Return	Maximum 3-Month Return	Minimum 3-Month Return	Maximum Trailing 12-Month Return	Minimum Trailing 12-Month Return
Cap Weight	**-0.94**	**1.39**	**10.11%**	**-16.05%**	**23.27%**	**-29.66%**	**38.96%**	**-41.75%**
Economic Scale	-0.76	2.23	16.01%	-14.94%	32.90%	-27.52%	46.38%	-44.08%
Global Equal Weight	-0.98	3.12	15.08%	-18.50%	30.26%	-32.27%	48.87%	-44.17%
Equal Weight Cap 1000	*-0.95*	*2.46*	*13.43%*	*-18.86%*	*28.41%*	*-33.08%*	*44.56%*	*-44.22%*
Equal Weight Econ 1000	*-0.77*	*3.66*	*18.63%*	*-18.47%*	*37.55%*	*-32.42%*	*54.96%*	*-45.01%*
Minimum Variance	-1.20	2.09	6.38%	-12.20%	15.61%	-20.42%	42.35%	-31.40%
Efficient Beta	-1.04	1.97	10.56%	-14.39%	22.44%	-25.94%	41.39%	-39.26%
All-Four-Combined	-1.04	2.25	11.69%	-15.42%	24.37%	-27.55%	41.48%	-40.51%

Sources: Research Affiliates, LLC, and Lazard Asset Management

The other surprise is the soaring efficacy of the combined strategies. Whether we choose efficient beta or all-four-combined, the statistical significance of the positive alpha sharply exceeds 99.9 percent significance, once we adjust for the value, size and momentum effects. Diversifying between different indexing strategies does not hurt performance, even—or perhaps *especially*—adjusting for these risk factors. At the same time, the HML loading of 0.31 in the case of efficient beta and of 0.32 in the case of all-four-combined contributes some of the incremental return, documented in the previous tables.

To model the impact of trade costs and market impact, we created a hypothetical $1 billion portfolio for each strategy as of Jan. 1, 2009 (see Figure 9). The average two-way turnover for the strategy was divided by the number of rebalance dates, and this percentage was used to create a proportional slice of the portfolio that was then assessed for commissions, fees and market impact. Trade costs were estimated using the ITG ACE model.[9] Estimated commissions, taxes and fees were then included to create a total estimated transaction cost. The resulting costs were then multiplied by the number of rebalance dates to determine annualized trading costs.

For the rightmost column, we modeled capacity using another simple set of assumptions. We assume that our portfolio will become difficult to manage—that tracking the intended "index" will be challenging—for any holdings that exceed 10 percent of current float or 10 percent of annual share volume. We then find the portfolio size at which 5 percent of the portfolio would be running up against one or another of these thresholds—i.e., would become "difficult." To our surprise, cap weight shows a "capacity" based on these limitations of less than $700 billion in size. The other core strategies would run into these same barriers at anywhere from $50 billion to a quarter-trillion in assets. These are not small sums. *But, we can probably infer that these thresholds are pretty conservative, based on the simple fact that cap weight is used to index vastly more than $700 billion in assets!*

Having the option of higher alpha and somewhat lower volatility is quite helpful for the average investor's portfolio. To see this, we plot on Figure 10 the minimum variance frontier of a diversified basket of various asset classes, where we compare the frontier with equity being represented by cap weight and efficient beta indexes. The optimized frontier with efficient beta providing equity exposure is clearly expanding the set of returns attainable to the investor. Over this history, at least, investors could achieve the same returns with lower levels of risk, or earn higher returns while keeping the same level of risk in the portfolio.

Intuitively, this is represented in Figure 11, where we plot excess annualized returns of the efficient beta over cap weight against excess volatility, on rolling three-year spans. Just as in standard mean-variance charts, the "northwest quadrant" is the preferred position for the investor—higher returns with lower volatility. This figure shows that selecting the efficient beta moves investors in the desired direction most of the time. With the caveats that our history is not terribly long and that past is not assuredly prologue, the portfolio never once offers higher volatility than cap weight, and delivers less return in less than 10 percent of the rolling three-year spans in our study, all centered on the peak of the tech bubble.

Conclusion

This study focuses on two interrelated pragmatic questions: If we want less risk, do we have to lower our equity exposure? Alternatively, can we achieve our intended long-term return goals with less in equities? Apropos of that simple question, our work explores the simple merits of diversifying our core portfolio—in effect, our beta risk. In a world in which many investors are considering ways to reduce portfolio risk, whether because of management pressure or because of fear of the consequences of misaligned risks, few would disagree that a bigger tool kit—a wider array of interesting alternatives—will be broadly welcomed. Some will choose to "de-risk" by lowering their equity market exposure, by aligning assets with liabilities, by reducing their active management risk *or by exploring ways to achieve better returns with similar risk in their core equity holdings.*

One can make a very good case that these strategies do not offer alpha, but offer "better beta." After all, none of these portfolios uses "stock selection" in any classical sense of the term. There are no interviews with management, no forecasts of future business prospects and no careful parsing of financial statements. In one case, we ask: "How big is the

Figure 8

Four-Factor Model In Global Universe, 6/1994-6/2009

Portfolio/Index	Beta	SMB	HML	Momentum	Monthly Intercept	4-Factor Alpha	4-Factor Info Ratio	Alpha *t*-Statistic
Cap Weight	—	—	—	—	—	—	—	—
Economic Scale	1.06	0.03	0.43	-0.04	0.17%	2.07%	0.78	3.17**
Global Equal Weight	1.06	0.18	0.36	-0.07	0.15%	1.82%	0.82	3.33**
Equal Weight Cap 1000	*1.04*	*0.19*	*0.05*	*-0.07*	*0.11%*	*1.27%*	*0.59*	*2.39**
Equal Weight Econ 1000	*1.08*	*0.24*	*0.47*	*-0.10*	*0.26%*	*3.08%*	*1.09*	*4.42***
Minimum Variance	0.82	0.02	0.50	-0.07	0.15%	1.81%	0.54	2.18*
Efficient Beta	0.96	0.02	0.31	0.01	0.11%	1.29%	0.88	3.57**
All-Four-Combined	0.98	0.06	0.32	-0.01	0.12%	1.42%	0.98	3.97**

* Significant at 95% confidence level. ** Significant at 99% confidence level.
Sources: Research Affiliates, LLC, and Lazard Asset Management

Figure 9

Estimated Transaction Cost					
Portfolio/Index	**One-Way Turnover %**	**Annual # Rebalances**	**Rebalance Trade Cost bps of Trade**	**Annualized Trade Cost bps of Portfolio**	**Capacity of Current Portfolio (in $billions)**
Cap Weight	**6.8**	**1**	**33**	**5**	**690**
Economic Scale	15.2	1	41	12	260
Global Equal-Weight	29.7	1	60	36	140
Equal-Wgt Cap 1000	*34.4*	*1*	*63*	*43*	*88*
Equal-Wgt Econ 1000	*28.9*	*1*	*60*	*35*	*80*
Minimum Variance	12.7	4	34	9	51
Efficient Beta	15.8	12	36	11	139
All-Four-Combined	21.1	12	43	18	113

Sources: Research Affiliates, LLC, and Lazard Asset Management

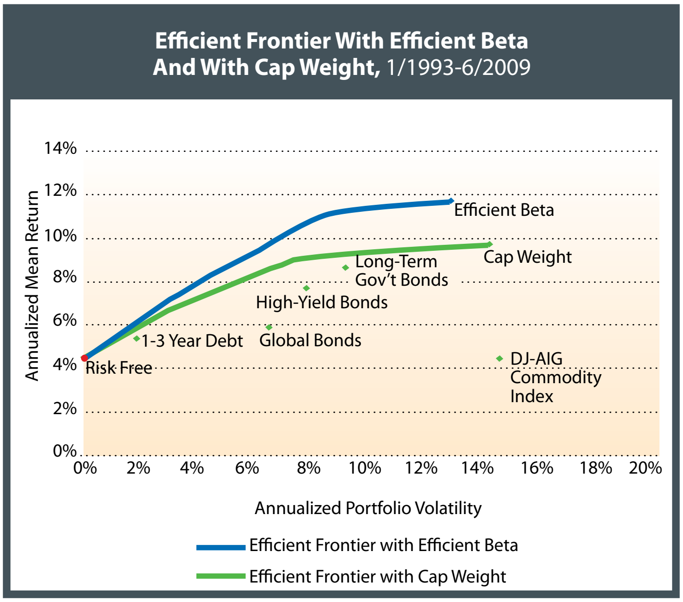
company's current book of business?" That defines both the selection and weight for the economic scale portfolio. In another, we ask: "Can we create a portfolio that is designed to achieve high risk-adjusted returns without the use of return expectations?" That defines the minimum variance portfolio. In yet another, we ask: "Why should we favor any stock over any other?" That leads to global equal weight, for which the only active decision—*a nontrivial decision!*—is to select the universe that we will equal-weight.

Our research shows that a combination of cap weight, economic scale and minimum variance creates a compelling risk/return profile. The purists will presumably argue that classical finance theory supports only one of these: cap weight; they would, of course, be correct. Others—all tacitly believers in some form of market inefficiency—might argue for one or another alternative to cap weight. Advocates of the status quo, with its singular reliance on cap weight for core indexed portfolios, will undoubtedly point to the fact that past is not prologue: "Past performance is no guarantee of future results."

Cap-weighted indexes are widely used; they are the generally accepted benchmark for gauging investment success. This simple reality creates "maverick risk"—a risk of underperforming our peers—for those investors who choose any of these alternative approaches, including a blended approach. Still, the selection of quasi-passive investment strategies within equities need not be limited to cap weight, nor need we forever rely on a cap-weighted benchmark. Perhaps investors can better serve their long-term needs by assessing which of these strategies—or combination of strategies—best conforms to their appetite for risk.

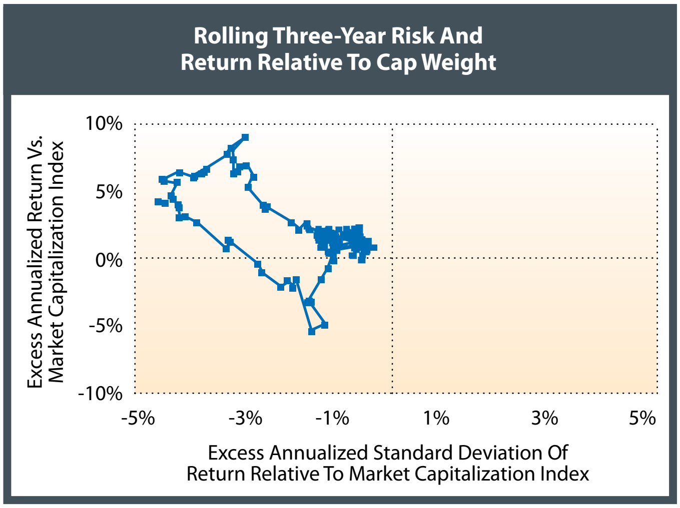
Fixed-income investors have a long history of considering risk and exposure when choosing the duration and credit of active and passive bond portfolios; broad bond-market index funds are far less widely used than stock index funds. Similarly, currency investors typically do not use "market cap" or even GDP as a guide for anything other than liquidity. Perhaps it's time to revisit our automatic reliance on cap weight as the sole strategy for measuring stock market suc-

Figure 10



Sources: Research Affiliates, LLC, and Lazard Asset Management

Figure 11



Sources: Research Affiliates, LLC, and Lazard Asset Management

cess, or as the default choice for our core equity holdings.

It is not our intent in this paper to explore the theoretical implications of our work, though we acknowledge that they may prove significant. These results—as with so many before—are not consonant, in aggregate across time—with an efficient market. The empirical results suggest some global inefficiencies that may prove to reflect an immense gap between expected risk and subsequent observed risk, or between expected return and expected utility, or between priced risk factors and the risk factors that *should* be priced in an efficient market. It is well beyond the scope of this simple empirical study to explore these nuances.

If an investor does not have a compelling view that favors one of these strategies over any other, then a diversified approach to beta can perhaps give us access to broad market exposure, without undue tilts to any single method, without undue reliance on market efficiency and with stronger empirical results than any single method.[10]

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Endnotes

1. In an effort to finesse the controversy regarding the terms "active," "passive," "index" and "strategy," as relates to noncap-weighted portfolios, we generally refer to all of these indexes, including cap weight, as "strategies" or "portfolios."
2. The economic scale approach uses the standard Fundamental Index methodology to determine the weights in an index. See Arnott, Hsu and Moore (2005) for details on the methodology. Fundamental Index® is patented and related labels and concepts are protected by trademarks, copyrights and patents, owned by Research Affiliates, LLC.
3. We cannot know "clairvoyant value" for any asset today. We can know the *past* clairvoyant value of most assets, especially if we go far back in time, so that the long subsequent history of distributions can be discounted back to an ancient starting date. For a detailed exploration of the nuances and surprisingly rich implications of clairvoyant value, see Arnott, Li, Sherrerd (2009a and 2009b).
4. Tacitly, this means that a forecast for the covariance matrix drives our portfolio construction. The "forecast" incorporated within the minium variance models relies purely on observed and actual historical data. No actual forward-looking forecast is embraced, at least none that are not included in actual long-term figures.
5. We incorporate a proprietary size factor that controls relative size exposure so the portfolio is not dominated by small, illiquid companies. We also impose constraints that limit GICS sectors to a maximum of 20 percent and individual securities to 1.5 percent, and an additional measure that moderates GICS industry group exposures to ensure that the portfolio offers a broad distribution of exposure across industries. We also restrict the investable universe by market cap and trading volume, to ensure that the strategy invests only in liquid stocks, as measured from the perspective of a large investor in this strategy. The market cap and volume constraints do not materially change the risk and return characteristics of the portfolio, but they do have a bearing on scalability and investability.
6. See Choueifaty and Coignard (2008).
7. This assumes independence of relative performance between years, which is a reasonable approximation of the observed empirical results.
8. The Fama-French factors are recalculated using cap weight portfolio, consisting of the 1,000 largest market-cap companies in the 23 countries in the FTSE and MSCI developed world indexes. To construct SMB and HML factors, we use MSCI Small Growth, MSCI Small Value, MSCI Medium Growth, MSCI Medium Value, MSCI Large Growth and MSCI Large Value, which we use to compute factor returns:

   SMB = 1/2(Small Value + Small Growth) - 1/2(Large Value + Large Growth)

   HML = 1/3(Small Value + Mid Value + Large Value) - 1/3(Small Growth + Mid Growth + Large Growth)

   To define momentum, we use cap weight portfolio, consisting of the 1,500 largest market-cap companies in the 23 countries in the FTSE and MSCI developed world indexes, which we sort monthly into three tiers of 500 stocks based on the prior return measured from month -12 to -2. Momentum return is the difference of returns of the top-tier equally weighted portfolio minus the bottom-tier equally weighted portfolio. Factor loadings of the portfolios are calculated based on a multivariate regression of portfolio returns against these factors.
9. The model incorporates stock-specific econometric models of volatility and price impact and provides the expected cost of trades as shown above. Its key inputs are stock-specific volatility, bid/ask spread, volume, closing price, intraday volume and volatility distribution as well as trade-specific size, side, strategy and expected time to completion. The model attempts to balance the competing forces of cost (spread cost and market impact) vs. risk (opportunity cost of uncompleted trades).
10. For a discussion of whether these non-cap-weighted portfolios are active or passive, or are indexes or strategies, please see the online appendix to this paper at www.journalofindexes.com

References

Ang, Andrew, Robert J. Hodrick, Yuhang Xing and Xiaoyan Zhang. (2006). "The Cross-Section of Volatility and Expected Returns," *Journal of Finance*, vol. LXI, no. 1, February: 259-299.

Arnott, Robert D. (2004). "Blinded by Theory: How Finance Theory Leads Us Astray," *Journal of Portfolio Management*, vol. 30, no. 5, Thirtieth Anniversary Edition: 113-123.

Arnott, Robert D., Jason C. Hsu and Philip Moore. (2005). "Fundamental Indexation," *Financial Analysts Journal,* vol. 61, no. 2, March/April: 83-99.

Arnott, Robert D., Feifei Li and Katrina Sherrerd. (2009a). "Clairvoyant Value and the Value Effect," *Journal of Portfolio Management*, vol. 35, no. 3, spring: 12-26.

———. (2009b). "Clairvoyant Value II: The Growth/Value Cycle," *Journal of Portfolio Management*, vol. 35, no. 4, Summer: 142-157.

Blitz, David C. and Pim van Vliet. (2007). "The Volatility Effect: Lower Risk Without Lower Return," *Journal of Portfolio Management*, fall: 102-113.

Choueifaty, Yves and Yves Coignard. (2008). "Toward Maximum Diversification," *Journal of Portfolio Management*, vol. 35, no. 1, fall.

Dorn, Daniel and Gur Huberman. (2009) "Preferred Risk Habitat of Individual Investors," Available at SSRN: http://ssrn.com/abstract=1100687

Falkenstein, Eric G. (2009) "Risk and Return in General: Theory and Evidence," Available at SSRN: http://ssrn.com/abstract=1420356

Haugen, Robert A., and Nardin L. Baker. (1991). "The Efficient Market Inefficiency of Capitalization-Weighted Stock Portfolios," *Journal of Portfolio Management*, vol. 17, no. 3, spring: 35-40.

Held, Jeremy. (2008). "Don't Get Mad, Get Even," *Institutional Investor Exchange Traded Funds Indexing Innovations, 7th Anniversary Issue*.

Hsu, Jason. (2006). "Cap Weighted Portfolios are Sub-Optimal Portfolios," *Journal of Investment Management*, vol. 4, no. 3, Third Quarter: 1-10

Lazard Quantitative Equity Team. (2009). "Managing Equity Risk–Risk and Reward Redefined," White Paper.

Markowitz, Harry M. (2005). "Market Efficiency: A Theoretical Distinction and So What?" *Financial Analysts Journal*, vol. 61, no. 5, September/October: 17-30.